What Happened To The Precision?

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Confidence Level

- This refers to the (1-2α)100% two-sided confidence interval for the true value.
- For symmetric confidence intervals, the relative precision is half the width of the confidence interval divided by the estimated value, expressed as a percentage.

Precision vs. Relative Precision

- Precision is a value calculated from the standard error of the estimate and a number based on the confidence level and probability distribution of the estimator.
- Relative Precision is the ratio of precision to the total.
- Precision and Relative Precision are estimated by substituting sample estimates for the theoretical values.

What Decreases Precision?

- Increasing standard error (standard deviation of estimator) by itself makes both precision and relative precision worse.
- Decreasing the absolute value of estimated tax due by itself
  - makes relative precision worse
  - does not affect precision.

What Increases Standard Error?

- Including positive and negative invoices sometimes increases standard error of estimation.
- Under sampling (N is too small) since standard error of estimation is inversely proportional to the square root of sample size.

What Decreases Total Tax Due?

- Fewer errors than expected.
- Errors that are randomly positive and negative so the net is “close” to zero.

Error Rate

- In this talk, error rate, p, is
  - The proportion of items in the sampling frame that have errors in tax paid. Here, p is usually small.
- This is usually unknown but may be estimated by:
  - A pilot sample
  - History of previous audits with this taxpayer
  - Experience of the CAS and/or Auditor

Known vs. Unknown

- Known:
  - File of invoice amounts (Sampling Frame)
- Unknown:
  - File of errors in tax paid.
- Sample size is calculated on the known information and assumptions about the relationship between the known and the unknown.

Expected Error Rate Model I

- Assumptions:
  - Errors are random and independent of the item being audited.
  - The error rate is constant over the entire sampling frame. (Note: Theoretically, a different rate could be applied to each stratum. I don't think it is worth the effort.)
  - All the errors are in the same direction.
- The last assumption is critical
  - For estimating taxability it is usually no problem.
  - For error in tax paid it may not be true.

Expected Error Rate Model I (Continued)

- This model adjusts the mean and standard deviations which are used in the formula for sample size.
- Model: W = XY where
  - X is invoice amount
  - Y is Bernoulli random variable with P(Y=1) = p and P(Y=0) = 1-p, $0 < p < 1$
- A detailed explanation of the model is included in the workshop package.

Expected Error Rate Model II

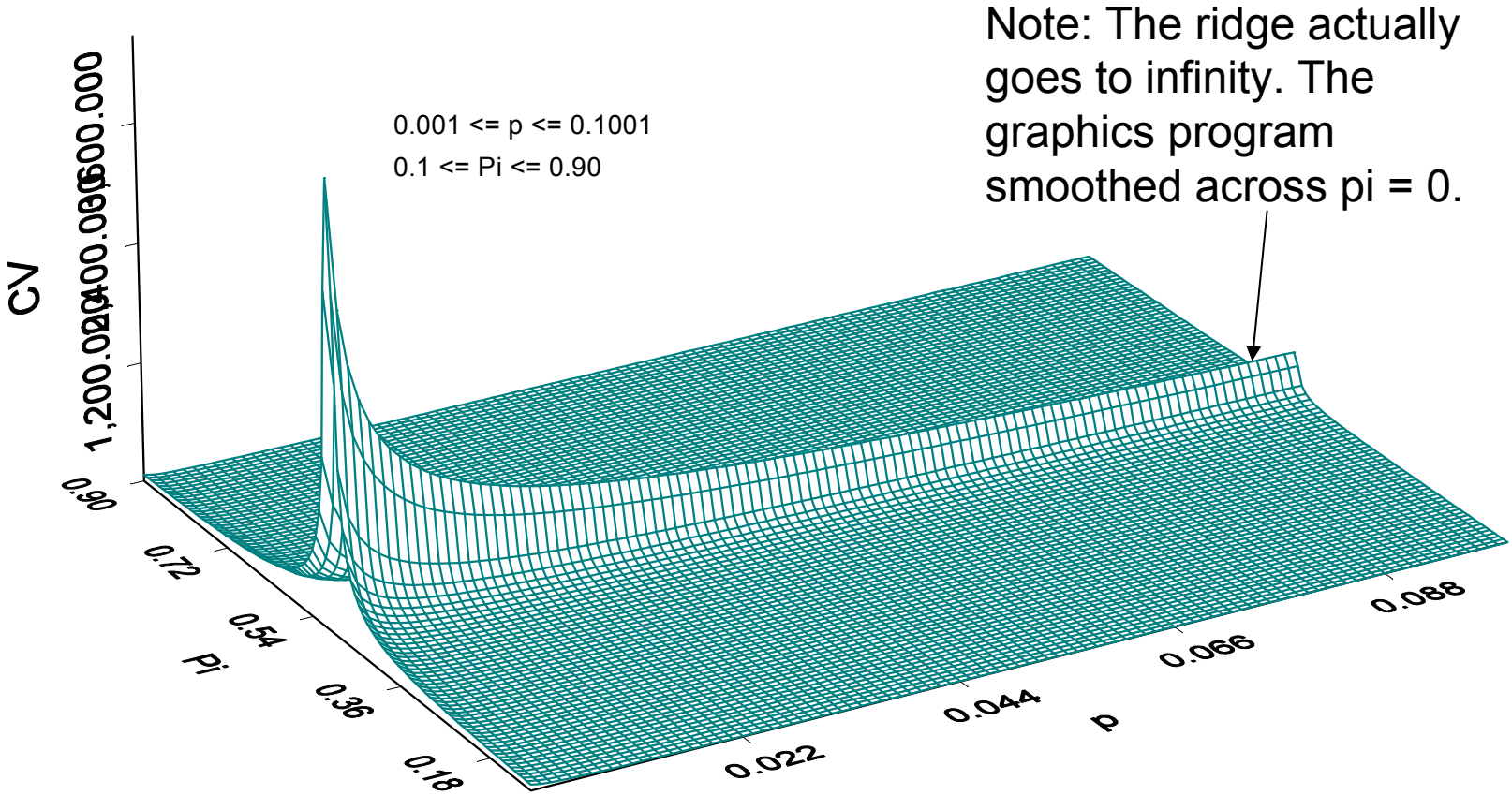
- All of model I plus an independent Bernoulli random variable indicating the direction of the error, positive or negative.
- Model: W = XYZ where
  - X is invoice amount
  - Y is Bernoulli random variable with $P(Y=1) = p$ and $P(Y=0) = 1-p$, $0 < p < 1$
  - Z is Bernoulli random variable with $P(Z=1)= \pi$ and $P(Z=0) = 1- \pi$

Implications of Model II

- When the errors are randomly positive and negative with $\pi = ½$ the theoretical average error is always zero, regardless of the average invoice amount, μ, or the error rate, p, for $0 < p \leq 1$.
- The coefficient of variation (cv) is the ratio of the standard deviation to the mean. When $\pi = ½$ it is unbounded.

Expected Error Rate Model II

Coefficient of Variation for ABC Aircraft Fabricators Data



Note: The ridge actually goes to infinity. The graphics program smoothed across pi = 0.

Practical Implications

- When the errors are all (or mostly) the same sign, increasing sample size can improve both precision and relative precision.
- When the errors are randomly positive and negative so that the net is “close” to zero, increasing sample size will not improve relative precision, but should improve precision.