SAMPLING POLICY
&
GUIDELINE MANUAL

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PREFACE

This manual is intended to provide guidance in sampling procedures for the Multistate Tax Commission (hereafter referred to as the "Commission"). It is also intended to inform the member states as to the sampling procedures the Commission's audit staff utilizes when conducting audits on their behalf.

It is assumed that electronic are available and statistical methods will be applied. However, under the right circumstances many of the procedures contained within this manual may be utilized when electronic records are not available. The Commission believes that statistical sampling should be used in most situations when electronic records are available and is the default procedure.

While it is not possible to fully explain all sampling procedures in this manual, we do hope that it will serve to augment what is covered in the Commission's sampling training classes.

In addition, the policies of this manual will be applied in any audit done on behalf of a member state only in the absence of any stated policy of that state. State adopted sampling policies, where they exist, will be applied if they are in variance with those expressed herein. Where no state policy exists, the Commission encourages its member states to establish sampling policies – hopefully in accordance with those expressed here.

General Layout – This manual is divided into six sections:

1. Section 0100 is an introduction
2. Section 0200 discusses characteristics of sampling
3. Section 0300 discusses statistical sampling procedures
4. Section 0400 discusses non-statistical sampling procedures
5. Section 0500 special auditing concerns relating to sampling
6. The final section are the appendixes A – G

Acknowledgements – All the content of this manual is the sole responsibility of the Commission. Some contributions were made by individuals and organizations outside the Commission. It is not possible to acknowledge all contributions. However, there are a few individuals that we wish to specifically name for providing us with critical background on the subject.

- Dr. Donald Roberts, University of Illinois for opening our eyes through his valuable instruction and his book, *Statistical Auditing* of which both proved invaluable.
- Dr. Allan Kvanli, University of North Texas for his assistance and guidance in developing and conducting our *Basic Statistical Sampling* manual and class. For his willingness to share his knowledge in answering our many questions.

INTRODUCTION

0101.00 Goal of Manual

This Sampling Policy and Guideline Manual (hereinafter referred to as "Manual" or "Sampling Manual") is intended to provide policies and guidelines to aid the Commission auditor in performing sampling in sales and use tax audits. On the other hand, this manual should not be used as a substitute for appropriate sampling training.

It is expected that before attempting to sample and implementing any of the policies and guidelines contained within this manual, the auditor will have already been properly trained in appropriate sampling techniques. The auditor is ultimately responsible for their work and therefore should consult with either their supervisor and/or Computer Audit Specialist (CAS) if there are questions regarding anything contained within this manual.

Failure to follow the policies and guidelines contained herein or lacking a thorough understanding of them may cause the Commission and/or taxpayer to waste resources.

0102.00 Goal of the Commission

The Commission endeavors to use electronic data for the purposes of sampling. Sampling is frequently necessary to realize efficiencies for both the Commission and the taxpayer. Sampling may minimize costs associated with the audit. The costs to retrieve and examine sample units affect both the taxpayer and the Commission. Therefore it is the goal of Audit Program to minimize sample sizes to the lowest levels.

In addition to being efficient, sampling should be as accurate as possible. Probability sampling can be used to provide information about the accuracy of an audit result based on a sample. Accuracy is greatly dependent on sample size. In

this way, there is a tradeoff between efficiency and precision. Probability sampling can be used to provide a balance between these objectives.

0103.00 Sampling Policy and Guidelines

It is recognized that each sampling situation may be unique. Guidelines are recommendations that should be followed, but can be deviated from based on the experience of the auditor or his/her supervisor. Departures from policies and guidelines should be documented and explained in the sampling plan, which will be contained within the completed work papers.

0104.00 When Should Sampling Be Used?

Note that the auditor, in determining whether error exists or whether further audit examination is warranted, may use sampling methods as they deem necessary. On the other hand, in considering sampling in supporting an audit finding that corrects or adjusts the tax amounts reported by the taxpayer, the Commission should consider the three STANDARDS discussed in this section.

BEST EVIDENCE STANDARD: The Commission auditor has the duty to use the best available evidence on which to base the audit finding that corrects or adjusts the tax amounts reported by the taxpayer.

A Commission auditor can come to an audit finding for the entire period based on an examination of all records, or an examination of part of the records (a sample). While both methods are acceptable, it is preferred under the best evidence standard, wherever practical, that an equal complete coverage of all records be made when this can be done given time and resource considerations. If the auditor believes that a detailed audited is warranted, the auditor is not obligated to sample irrespective of the taxpayer's preferences. It is not necessary for the auditor to obtain agreement from the taxpayer as to whether a detailed audit is performed.

However, as a matter of fact, most audits done by the Commission, it is not feasible to do a detailed examination of all records. Commission audits will involve multi-state taxpayers with voluminous and complex records, necessitating sampling procedures. In applying sampling, the Commission should use the **STANDARD TO JUDGE AN AUDIT FINDING BASED ON A SAMPLE** to accept the results of a sample. Under this standard, the result from a sample is acceptable if within an acceptable degree of accuracy, using a reasonable degree of confidence, the same result would have been found had an equal, complete, and detailed audit for the entire audit period for all transactions of audit interest been done. As with a detail examination, the auditor can use the results of a sample whenever this standard is met. However, it is prudent to consider and respect any reasonable requests made by a taxpayer with respect to sampling.

Probability samples and judgmental samples are two acceptable types of samples used in audits for acquiring sufficient audit evidence upon which the Commission auditor can base their audit finding. The preferred sampling method is use of probability sample using statistical sampling procedures applied to all transactions under the scope of the audit objective for the entire audit period. Use of a probability sample using non-statistical sampling procedures is also acceptable, but not preferred. Any sample other than a probability sample is considered a judgmental sample, and may be acceptable under certain circumstances where a probability sample is not possible.

The sampling policy of the Commission is to do a probability sample using statistical sampling procedures when possible and practical given the **BEST EVIDENCE STANDARD** and **STANDARD TO JUDGE AN AUDIT FINDING BASED ON A SAMPLE**. It is possible to compare the audit results from a probability sample to a 100% examination (had one been preformed). This is not possible with a judgmental sample unless a 100% examination is later done. Therefore, under the best evidence standard and standard to judge an audit finding based on a

sample, the Commission auditor is obligated to take the probability sample if one is practical under the circumstances. In all other cases, the Commission auditor may take a judgmental sample, subject to the limitations and guidelines expressed in this manual.

NOTIFICATION AND DISCLOSURE STANDARD: The Commission auditor is obligated to disclose, prior to sampling, the method of sampling to be used to obtain a result, regardless of whether a probability or judgmental sample is considered. Further, the Commission is obligated to disclose the results of a sample to the taxpayer, regardless of whether a probability or judgmental sample was taken.

The auditor is obligated to record and report any known objections the taxpayer has in sampling procedures.

0105.00 Objectives

This manual covers the basic procedures for planning, conducting and documenting a sample that utilizes the most appropriate sampling techniques after considering the audit circumstances and has the following objectives:

- To help the auditor obtain appropriate information about the taxpayer prior to conducting a sample.
- To help the auditor collect, organize and present all information, which is pertinent to the performance of the sample.
- To inform the taxpayer of the Commission's requirements in the conduct of a sample.
- To encourage cooperation between the taxpayer and auditor.

0106.00 Responsibilities

In any Computer Assisted Audit (CAA) the chances of success are greater if auditor and the Computer Audit Specialist (CAS) communicate and work together in devising a sampling plan. The auditor should take under serious consideration the

sampling advice provided by the CAS; otherwise the sampling effort might not come to a successful result. Most importantly, the taxpayer must be informed about the sampling process. Care should be taken to involve the taxpayer in the process as soon and as much as possible.

Probability sampling can be performed in a CAA or in any other audit. A CAA is an audit in which the taxpayer is able to provide the auditor with electronic records. This includes electronic records that may be provided in text form, spreadsheet, and databases or downloads from main-frame computers.

The primary responsibility of any audit lies with the auditor who has the audit assignment. However, this should not preclude the auditor from seeking the advice and assistance from the CAS when conducting a CAA or the use of statistical sampling.

0106.10 Responsibilities of the Auditor

1. Discussing the sampling plan with the taxpayer.
2. Providing the CAS with details on items in the population to be excluded from the sampling frame.
3. Establishing dollar cutoffs (low dollar items not audited and high dollar items looked at in detail).
4. Determining items reviewed on a detailed basis.
5. Securing and reviewing the sampling units.
6. Valuing each sampling unit for error.
7. Providing the audit conclusions to the CAS for a statistical evaluation.
8. Projecting the sample results in the audit report.
9. Discuss the sampling results with the taxpayer.
10. Answer any questions the taxpayer may have regarding the results of the audit.

0106.20 Responsibilities of the CAS

The specialized nature of statistical sampling requires specific guidelines and policies (see section 0400). One of the responsibilities of CAS is to provide guidance to the auditor regarding methods on probability sampling. In addition to designing statistical samples in CAA audits, the CAS is responsible for:

1. Assisting the auditor in answering taxpayer questions about statistical sampling.
2. Meeting with auditors, taxpayers, and consultants regarding sampling.
3. Informing audit management and other interested parties of the consequences of sampling policy.
4. Representing the audit section on issues involving statistical sampling in contested audits. When the procedures outlined in this manual are followed, this responsibility applies whether or not CAS was actually involved in the design of the sample.
5. Discussing the sampling procedures with the taxpayer and auditor.
6. Assist the auditor in defining the sampling frame from the data provided.
7. Giving advice on the appropriate sample size.
8. Assist the auditor if needed in stratifying the sampling frame.
9. Assist the auditor if needed in selecting a probability sample from the sampling frame.
10. Assist the auditor if needed in evaluating the audited sample results for any statistical projections.
11. Providing the auditor with suggested text for the audit narrative concerning the sample.

0107.00 Auditor Training

The audit director and the auditor's supervisor are ultimately responsible for insuring that the auditor is properly trained and has demonstrated an

understanding of the sampling techniques employed by the Commission before allowing the auditor to conduct a sample of any nature.

0108.00 Deviations from Manual

The auditor-in-charge may deviate from the guidelines with the permission of the audit supervisor in order to adapt to the particular circumstances of the audit. Before permission is granted on deviations from the policies, the audit supervisor should first consult with the CAS and audit director.

0109.00 Revisions of Manual

This manual may be revised from time to time. Revision dates will appear at the bottom of revised pages.

SAMPLING CHARACTERISTICS

0201.00 What is Sampling?

Sampling is "the application of an audit procedure to less than 100 percent of the items within an account balance or class of transactions for the purpose of evaluating some characteristic of the balance or class."

A sample can be drawn many different ways, but principally there are two basic types of samples. An objective form of sampling involves selection based on chance, where the probability of selection is known for each item in the population, but not necessarily equal with respect to all sampling units in the population. This type of sampling is called probability sampling. All other forms of sampling other than a probability sample are considered judgmental samples. Probability samples include simple random samples and stratified random samples. Judgmental samples include large block samples that have been common in sales and use tax auditing.

0202.00 General Reasons for Sampling

In general, an auditor will sample in order to reach a conclusion about taxable amounts in a population that is too large to examine in detail. Sampling is frequently necessary to realize efficiencies for both the Commission and the taxpayer. Sampling may minimize costs associated with the audit. The costs to retrieve and examine sample units affect both the taxpayer and the Commission. Other reasons can include:

0202.10 Estimation

The auditor wishes to estimate the unknown value of tax error. This usually is the main objective for sampling.

0202.20 Prevention

If a probability sample of all transactions covered by the audit objective for the audit period is utilized in auditing, the audited party cannot predict which items will be chosen for review. In this way, incentive is given for improved tax compliance.

0202.30 Detection

Generally, it is not known whether tax errors exist. Sampling can be used to detect the presence of errors.

0202.40 Cost Benefit – Auditor Efficiency

We sample to achieve greater auditor efficiency. Auditor time is expensive and scarce. A detailed examination is often unreasonable or impractical due to the complexity and volume of taxpayer records.

0202.50 Taxpayer Efficiency

Sampling can make the audit process less intrusive for the taxpayer. It may result in less audit time in the taxpayer's office and fewer records to provide for review.

0203.00 Justification for the Sampling Procedure Used in the Audit

The three standards expressed above, will dictate which method of sampling is appropriate. No one sampling technique is best suited for all audits and a proper determination cannot be made until there is a thorough understanding of the taxpayer's operations. Of course, auditor judgment is relied upon in determining how the three standards apply to any audit. As such, the auditor should be allowed discretion in determining sampling procedures. However,

having used the latitude expressed here, the auditor must be able to justify the procedures used in the audit, in light of the three standards.

If judgmental sampling is used the auditor will include in the audit work papers an explanation as to why it was determined this type of sampling was used. Similarly, use of non-statistical sampling procedures applied to probability samples also needs to be justified.

0204.00 Non-statistical versus Statistical Sampling

Non-statistical sampling procedures must be applied to judgmental samples. Although not preferred, non-statistical methods also can be applied to probability samples. A "statistical sample" is a probability sample that has been objectively evaluated using statistical methods (probability theory). The International Federation of Accountants (IFAC), an international organization of national accountancy organizations, has developed pronouncements called "International Standards on Auditing" (ISA) on various auditing topics, including sampling. A useful definition separating the two types of sampling can be found in ISA 530:

> Statistical sampling means any approach to sampling that has the following characteristics:
> (a) Selection of a probability sample; and
> (b) Use of probability theory to evaluate sample results, including measurement of sampling risk.
>
> A sampling approach that does not have characteristics (a) and (b) is considered non-statistical sampling.

Statistical sampling procedures are preferred over non-statistical sampling procedures. Even so, application of non-statistical procedures to probability samples can be justified under certain circumstances. In some cases, a probability sample can be taken, however, because it is expected that a large sample still will not provide an audit results that meets the **STANDARD TO JUDGE AN AUDIT FINDING BASED ON A SAMPLE**, the probability sample may be evaluated using non-statistical methods.

STATISTICAL SAMPLING PROCEDURES APPLIED TO PROBABILITY SAMPLES

0301.00 When to Use Statistical Sampling – Large Populations

In Commission audits, according to the standards expressed in section 0104.00, unless impractical, a detail is always preferred. Where a detail is not practical, the Commission auditor must either ignore certain audit populations, or consider some form of sample. Generally if a detail is not possible, statistical sampling is the sampling method of choice when the sampler wants information about the accuracy of the sample results.

When coverage of audit populations is considered through sampling, the standards in 0104.00 provide that the Commission auditor do a probability sample where practical. Statistical evaluation of the results is preferred. However, the standards recognize that there are times where it is not appropriate or practical to consider a statistical evaluation when a probability sample was taken.

In determining how to audit, the Commission auditor has five general ways of dealing with a population (in order of preference):

1. Detail examination,
2. Statistical sampling (probability sampling with a statistical evaluation – covered by sections 0301.00 – 0309.10),
3. Probability sampling without statistical evaluation (covered by sections 0401.00 – 0410.00),
4. Judgmental sampling (including block sampling), or
5. No audit coverage - most often this will be under extenuating facts that require no specific investigation of the detail (for example, the taxpayer makes all sales in a division of one product that is always considered

statutorily exempt from tax – a sales tax verification of the detail transactions is unnecessary in that division).

Any of these approaches may be valid under a certain circumstances. The standards in section 0104.00 determine the approach and the auditor must use their judgment in determining how these standards apply.

But as a practical matter, it is presumed that most audit populations in Commission audits will be **large**, requiring statistical sampling. This is because Commission audits are generally on large companies, with numerous transactions recorded in the accounting records, and where accounting records will be maintained in an electronic format. It is further presumed that if electronic records are kept, that they can be made available to the Commission in electronic format. Where audit populations are not large and/or not in electronic format, something other than statistical sampling will generally be considered under the standards in section 0104.00. These circumstances should be identified in the audit report.

What is considered "large" will differ from audit to audit. But this **most often** is a population of at least 2,000 – 5,000 sampling units given the circumstances at hand. This number range includes only those items that will be sampled excluding items that have no chance at being selected into the sample (such as high dollar items or other items removed from the population). Any sampling frame under 2,000 is generally too small to apply statistical sampling (although there may be exceptions). Any sampling frame over 5,000 sampling units will be considered "large". But as a practical matter, certain sampling frames that are between these amounts need to be analyzed to see if it is practical to apply statistical sampling. To determine if sufficient numbers exist for statistical sampling, separate criteria should be applied to stratified and un-stratified sampling frames.

Stratification is preferred unless extenuating circumstances exist that justify a un-stratified frame. When statistical sampling is considered, the auditor should generally look to stratify the frame whereby at least two strata will be sampled. For stratified sampling frames, the number of units available for sampling should be of sufficient size where:

1. Stratification is possible under the techniques described herein (see sections 0306.00 to 0306.40),
2. A sample size of at least the minimums can be taken from each of the strata (see section 0307.10),
3. It is possible that at least two of the strata can be sampled where sample size does not exceed 50% of the total sampling units available in a stratum, and
4. There is a reasonable chance that a precision goal can be met (see section 0308.70).

Populations that do not meet all these criteria are not candidates for statistical sampling unless it is practical to use an un-stratified frame. Similarly, populations under 2,000 sampling units that meet these conditions can be statistically sampled. Also, populations over 5,000 sampling units that do not meet these conditions will not be candidates for statistical sampling.

Statistical sampling techniques can be applied to sampling frames sufficiently large such that in an un-stratified frame the following conditions are all satisfied:

1. Sample size is at least 300, and
2. There is a reasonable chance that a precision goal can be met (see section 0308.70).

0302.00 Sampling Frame and Stratification Guidelines

Things to consider when developing and stratifying sample frames:

- In statistical sampling, the unbiased selection of the sampling units is important. Judgment is removed in the selection process. On the other hand, experience, intuition, and judgment are effective in refining and stratifying sampling frames. [1]
- One of the best strategies to provide better precision is to remove items that have little or no potential for material error.
- Stratification of the sampling frames often leads to better precision.
- Every time a projection is made, sampling error will result. Optimally, as few projections as possible should be used to minimize overall sampling error. However, there are frequently times where many individual projections are required.[2]

0303.00 Handling of Negatives

This section deals with the handling of "negatives" – meaning accounting entries that correct, change, or cancel a business transaction's recorded value in the books and records of the taxpayer. It also includes the accounting entries that reclassify various transactions within the records by transferring recorded amounts in accounts.

Note that the auditor will usually take a sample to determine total taxable error for a target population. For each sample unit, the auditor might have some accounting value, such as an invoice amount, book value, or other recorded amount. The auditor will examine the selected sampling units, and will in most cases, establish a taxable error value for each unit. The meaning of "negative" as used in this section does not refer to tax overpayments that may be present in the valuation of taxable error values established by the auditor.

In taking a sample, the auditor should sample from only the "positive" amounts. Although negative dollar amounts will not be directly sampled, they must never be ignored in the sampling process. Any negative that relates to a transaction that is selected into the sample should be reviewed and considered in the valuation of sampling units. Also, extraneous positive and negative items that zero out (when items related to the same transaction matched together total zero) should be removed from the sampling frame before sampling. Further, some negative items may be found that partially reduce corresponding positive amounts – these can be replaced by a net positive amount in the sample frame. The process of matching and removing related sampling units that total zero should be done **PRIOR** to making further refinements to the sampling frame. Note that the more complete and extensive the download from the taxpayer, the more likely that matching positives and negatives will be found. Normal procedures for handling negatives in the sampling frame are as follows:

1. When the download of transactions contains negative items, every reasonable effort will be made to match them with the corresponding positive item given the information available to the auditor. Ultimately the **ability** to match negatives with corresponding positives is the responsibility of the taxpayer in that the taxpayer is required to maintain adequate records and information on any business transaction that enables the auditor to match related accounting entries. The auditor is responsible for performing this procedure once sufficient information is obtained from the taxpayer. The auditor should request all data elements to accounting entries that enable the matching process to be done. The auditor is responsible for identifying any elements contained in the accounting records. Naturally, the taxpayer is responsible for disclosing all types of data elements that are maintained in the records to

the auditor, when a request is made by the auditor to provide a description of the accounting data.

2. Remove all zeros and negative items from the file that represent canceled transactions. If possible, substitute several positive and negative accounting entries with one sampling unit, if the net amount is a positive value and taken together, these accounting transactions all refer to the same business transaction within the target population.
3. Determine any low and high dollar cut-off amounts (when applicable) and remove these from the sampling frame. The high dollar amounts will be examined in detail.
4. Stratify the remaining positive values.
5. Determine sample size and allocate sample to each strata
6. Draw sample from each stratum
7. At this point the taxpayer will be given the sample items and given the opportunity to match any remaining negative items with any of the sample items drawn.

It is often difficult for some to understand why – as a matter of procedure – that no sample is drawn from the negatives. This failure to understand frequently arises for two reasons. First, it may not be immediately apparent sampling from negatives can cause selection bias. And secondly, there is the appearance that part of the accounting records will not be covered adequately for audit purposes if negatives are not directly sampled.

The nature of typical accounting files and inclusion of the negatives as sampling units may bias the sample selection process. Accounting files represent a target population of tax transactions and their tax error values. (Note that the tax error values can be either positive or negative.) Accounting files often do not provide for “clean” representations of target populations.

A sampling frame that corresponds to the target in such a way that each sampling unit refers once, and only once, to a specific and discrete part of the target population is optimal for sample selection purposes. Here, "discrete" means a separable and unique part of the target population. Therefore, when a sampling unit is randomly selected into a probability sample (without replacement), that portion of the corresponding target has only one chance at being selected into sample along with all other discrete portions of the target. However, if more than one sampling unit relates to a portion of the target population – that part of the target is more likely to be selected and valued for error when compared to the planned probability of selection for that discrete portion.

Accounting adjustments (negatives) will almost always (if not always) represent a transaction in the target that was previously represented by a positive. No negative transaction has ever initiated a sale or purchase, two of the most common target populations sampled in Commission audits. Note that some transactions in the target population may have no corresponding negatives, while others may have one or more corresponding negatives.

All efforts should be made to avoid bias in the sample selection process. Inclusions of the negatives may lead to such a bias where discrete portions of the target have varying probabilities of being selected that are not equal to the planned probabilities of selection. Selection bias cannot be easily cured or handled after the sample is drawn. Random selection from only the positives – after other extraneous positives have also been removed – provides a means of identifying the discrete portions of the target that should be valued for error.

Note that in some cases, it is difficult or impossible to remove extraneous positives (or even negatives when sampling from non-electronic files). If extraneous positives (and/or negatives) still exist in the sampling frame, a **decision rule** should be implemented as described in section 0504.00. An

extraneous positive is where more than one sampling unit with positive accounting values exists in the sampling frame and relate to the same discrete part of the target population.

0304.00 Defining the Audit Objective & Refining the Population

When performing an audit examination on a target population, the auditor should have a clearly defined audit objective (sometimes the auditor may have multiple objectives). Stating this objective can enable the auditor to identify the best sampling frame that covers the target population. Based on the objective, the auditor can also employ further refinement procedures that will minimize sampling error.

An attempt should be made to remove items that have little or no potential for error based on the audit objective.[3] This can be done by many different means:

- Removing transactions outside the audit period.
- Identifying transactions that are in the accounts of interest.
- Identifying transactions with pertinent cost codes, locations or cost centers that are being audited (such as excluding out-of-state locations or cost centers).
- Identification of certain vendors for exclusion.

When refining the sampling frame, the auditor should discuss these procedures with the taxpayer. The auditor should make all efforts to reach an agreement on what portions of the download that will be sampled. If agreement is not reached, generally those portions should, in most cases, remain in the sampling frame, unless it is known that inclusion will seriously impact sampling error and these items will likely not be material to overall results, if included in the sampling frame. Note that where it is known that

errors are very rare and will likely have little impact in the final results if found ("needles in the hay-stack"), inclusion of those portions of the download could cause sampling error to be much larger than what would have been had these portions of the download had been removed. For these reasons, Commission auditors will not always be obligated to construct sampling frames upon which complete agreement is reached with the taxpayer.

0305.00 Determination of Cutoffs

Consideration should be given to removing very high and low dollar amounts from the sampling frame. The auditor should make all efforts to reach agreement with the taxpayer in setting these limits.

0305.10 High Dollar Cutoff – "Ceiling"

Coverage of high dollar amounts on an actual basis is an effective strategy in providing improved precision. Removing the high dollar amounts from the sampling frame will reduce the range of dollar values in the non-zero errors. This will provide for a lower standard deviation in the errors, and therefore greater precision of the estimate.

Practically, consideration should be given to how high of an invoice amount could be projected to the entire population. This question can be used in setting the upper dollar limit. What is the highest dollar value amount the taxpayer would be willing to project? Consideration should also be given to how many items can be practically examined on an actual basis. Setting the upper limit too low will be inefficient. No projection will be made with high dollar items reviewed on an actual basis.

0305.20 Low Dollar Cutoff – "Floor"

Similar to high dollar cutoffs, a low dollar cutoff can also be established. The difference is that the low dollar invoices are generally ignored altogether. If a low dollar cutoff is set, the auditor should consider the total dollars that would be ignored in the invoices from $0 to the low dollar cutoff. Setting the low dollar cutoff too high will defeat the purpose of sampling.

Materiality plays an important consideration when setting a low dollar cutoff. It should be remembered that a $2.00 transaction may appear immaterial when examined. However, that single transaction in a probability sample may represent thousands of transactions and hundreds of thousands or even millions of dollars in some populations. The auditor should do some analysis of the data before making a determination as to where the low dollar cutoff should be.

One such analytical test is a "*what if test*". An example would be, if the error rate was "x%" on all transactions between $2.00 and $3.00 and that was projected against the population of transactions within those boundaries what is the potential tax impact? When doing such a test, good auditor judgment should be used in selecting an assumed error rate. Prior audit results may be useful in making such a determination. If the amount in question is determined immaterial or there is not a reasonable cost benefit ratio, then the auditor should consider using this as the low dollar cut off amount.

0306.00 Stratification

Stratification of the sampling frame normally leads to better precision when compared to a simple random sample of equal size for the entire sampling frame. Substantial gains in precision will generally result by adding a few

strata. But at some point the gains in efficiency decrease when adding more than just a few strata. Also, in some rare situations, stratification does not provide for a material improvement of relative precision.

Stratification in terms of dollars is used to reduce the effect of extreme values (errors). By reducing the variability of the population within each stratum, stratification often reduces the sample size needed to achieve a desired level of precision and reliability. Unless a population is homogeneous, stratified sampling is the Commission's preferred method to be used.

0306.10 The Number of Strata

The range between the low and the high dollar value items in the sampling frame is the biggest factor in determining the number of strata in the sampling frame. Usually, this will be anywhere from 2 to 6 strata (excluding the detailed or high dollar value stratum). By setting up more than four to five strata that will be sampled, only small gains in efficiency are generally realized.

0306.20 Strata Boundaries

There are a few rules that must be applied when employing stratification:

1. Every unit of the sampling frame must belong to only one stratum
2. The characteristics that define the strata should be made prior to sampling, (this is usually the dollar range of the stratum) and
3. The exact number of units within each stratum must be known prior to sampling

0306.30 The Types of Characteristics Used in Stratifying

Strata may be set up generally in one of two ways:

1. By a quality, such as location or time period (generally referred to as "grouping" rather than stratifying), or
2. By a quantity such as invoice amounts or GL distribution amounts.

306.40 Stratifying On Amount

Generally stratification is based on invoice amount. This can be done through one of two methods:

1. Cumulative Square Root of the Frequency Method
2. Proportional

Of the various methods available, the Commission has chosen the *cumulative square root of the frequency* method. Refer to Appendix D for a detailed explanation. The method is well documented in statistics references and is used by a number of taxing authorities. This method allows for greater efficiency in the sampling process compared with setting strata boundaries using judgment only.

Using judgment to determine the strata break values is a perfectly acceptable and will not invalidate the sample or its results, but may not be the most efficient method.

307.00 Sample Size

Statistical sampling differs from non-statistical sampling in several ways. One important difference is that sample size can be determined based on prior knowledge. This knowledge can be exploited to set sample size at a point where a sufficient degree of accuracy is attained.

Formulas may be used to statistically arrive at sample size based on this prior information. This information includes the estimated standard deviation and average of taxable the error value in the population. Two other factors that influence sample size is the desired confidence level and precision. Unfortunately, much of the time information concerning the average and standard deviation of the taxable errors is either not available, too impractical to obtain in advance, or simply just a guess (that can be widely off the mark). Absence of this information does not preclude statistical sampling or arriving at a statistically based sample size as the auditor can make certain assumptions that replace this information (however, these assumptions may not hold true in the final sample results).

On the other hand, a simple random sample or a stratified random sample may be statistically evaluated even if sample size was initially arrived at using non-statistical means. The International Federation of Accountants standard on sampling, ISA 530 discusses sample size:

> *When applying statistical sampling, the sample size can be determined using either probability theory or professional judgment. Moreover, sample size is not a valid criterion to distinguish between statistical and non-statistical approaches.*

In deriving sample size, the Commission auditor can use any of the following methods:

1. Using classical statistical formula that estimate sample size based on prior information (typically this information is gained from a probe sample) (section 0307.20).
2. Using statistical formula based on the error-rate model that substitutes some of the needed prior information with assumptions or knowledge concerning the rate of occurrence of the non-zeros in the population (section 0307.30).
3. An attribute method that attempts to observe a minimum number of nonzero errors in the sample given a error rate (section 0307.40).

4. A negotiated sample size with the taxpayer.
5. Considering what the largest sample that can be practically done with the time and resources available.

Any of the above methods may be used, provided they arrive at a sample size that is equal to or greater than the required minimums discussed in section 0307.10.

0307.10 Minimum Sample Size

At no time shall sample size be less than 300 sample units for unstratified sampling (one stratum overall that is sampled).[4] In stratified samples, no stratum will have a sample size of less than 100. In most stratified sampling frames, the sample size will be approximately 200 or more per stratum depending on the desired relative precision.

0307.20 Classic Formulary Approach

The classical formulary approach requires the following to arrive at a statistically based sample size:

1. An estimate of the standard deviation of the *error* amounts in the sampling frame,
2. An estimate of the average error amount in the sampling frame,
3. The desired relative precision, and
4. The confidence level.

Unfortunately, the first two pieces of information are very difficult to arrive at without performing a pilot sample. Pilot samples are frequently impractical. It is possible that the standard deviation and average taxable error computed in the probe sample may be significantly different than what will be encountered in the sample, meaning material over or under sampling is possible even when a probe sample was used.

Note that there is also a rule of thumb concerning sample size and relative precision:

> To achieve a two-fold increase in relative precision, such as when 15% is desired and 30% was attained; the sample size needs to be quadrupled. For example, if the sample was initially 800, it likely has to be set at 3,200 in order to attain desired relative precision.

The confidence and precision levels used to estimate sample size should be at the same that will be used in the final evaluation of the sample results. Most often, this will be at the stated levels shown in sections 307.6 and 307.7, unless superseded by state policy.

0307.30 Error Rate Model

Instead of taking a probe sample, which is generally required in the classic formulary approach (section 0307.20), the required information, the average and standard deviation, can be estimated using an estimate of the error rate in the population. The error rate model requires information as follows:

1. An estimate of the error rate (rate of occurrence of nonzero taxable errors),
2. The standard deviation of the *invoice* amounts,
3. The average of the invoice amounts,
4. The desired relative precision, and
5. The confidence level.

An educated guess can be made as to the error rate. In an electronic audit, the standard deviation and average are known (in a manual audit, these can be estimated).[5] Commission policy sets the relative precision and confidence level.

The auditor may make an estimate of the error rate through use of a pilot sample. Most of the time this is not practical and history or auditor

judgment must be used to estimate the error rate. If the error rate is set using judgment, a low error rate should be considered. However, error rates below 2% typically produce impracticably large sample sizes. If there is no history, sample size can be set using a 2% rate of occurrence standard.

0307.40 Attributes Method

Sample size may be determined using attribute techniques. This methodology attempts to detect a minimum number of errors with a stated confidence level for a given error rate in the population. Using this approach, precision is not considered. Tables, based on the hypergeometric distribution, exist that allow the auditor to compute these sample sizes. The minimum number can be based on the Commission policy of 3 errors (section 0308.80), or some other number greater than this.

0307.50 Sample Allocation in Stratified Sampling

Once the number of strata and strata boundaries has been determined, the overall sample must be allocated to the strata. The primary consideration is that sample size be sufficient within each stratum to find and weight errors if they exist. Two acceptable methods of sample allocation are:

1. Proportional or
2. Optimal

The preferred method used by the Commission to allocate samples is the optimal basis. See Appendix E

0308.00 Evaluation & Projection of Sample Results

If the minimum number of errors (3 per stratum) is observed in a probability sample designed by the CAS and auditor, a statistical evaluation should always be done. These results should be disclosed to the taxpayer, whether or not an actual projection is made.

0308.10 The Four Estimators

In projecting a total error amount, four estimation procedures are available. The first two, mean-per-unit and difference, measure a certain value for each unit in the sample. The other two methods, ratio and regression, measure a relationship between two numbers found for each sample unit. All four estimation methods will be calculated in statistical evaluations.[6]

0308.10.10 Mean-per-unit Estimation

Mean-per-unit estimation uses the audited values found in the sample to estimate the total audited value. Using an estimate of the total audited value, an estimate of total taxable error is made. The overall variance of the estimated total audited value is typically large for low error rate populations. Consequently, this method is useful only in high error rate populations.

0308.10.20 Difference Estimation

Difference estimation measures the taxable errors in the sample to arrive at the estimated total taxable error. Difference estimation ignores the invoice amounts or book values of the sample units. Frequently, this method provides results similar to the ratio and regression estimators, but usually is slightly less precise.

0308.10.30 Ratio Estimation

In sales and use tax auditing, the overall relationship between the error values and the invoice values is used to project the total error. There is generally some correlation between the invoice amounts and the taxable error values in typical populations sampled.[7] When some degree of correlation exists, ratio estimation often provides for better precision than either mean-per-unit estimation or difference estimation.

0308.10.40 Regression Estimation

Like ratio estimation, regression estimation measures the relationship between the invoice amounts and taxable error values found in the sample to arrive at a total estimate of total taxable error. Regression estimation provides a linear measurement of this relationship, and is usually the most precise estimator.

0308.20 Restrictions for Ratio and Regression Estimators

When an estimator is biased, this means that over all possible samples of a given size, the average value of all the estimates does not equal the actual population value. In an unbiased estimator, the two values will be equal.

The mean-per-unit and difference estimators are unbiased. Ratio and regression estimators have some bias. However, because of the expected gain in precision coming from ratio and regression estimators, these two methods are frequently preferred, despite the bias.

There are statistical concerns[8], including estimator bias, that preclude the use of ratio and regression estimation when one or more of the following occurs in the sample:

- The coefficient of variation (cv) for the estimated invoice amounts is over 10%
- The coefficient of variation for the estimated audited amounts using either the mean-per-unit or difference estimator exceeds 10%
- The overall sample size is less than 100

In computing the cv values above, any detailed stratum is ignored. None of these conditions preclude the use of mean-per-unit and difference estimation. Finally, the ratio estimator should not be used when the invoice values include both positive and negative values.[9]

0308.30 Combined Versus Separate Estimation for Ratio and Regression Estimation

In stratified random samples, ratio and regression estimation may be applied using two different approaches: "separate" or "combined". The Commission generally will use the combined approach only if the ratio or regression method is used to estimate total error (unless the Commission and taxpayer both agree to use the separate approach in addition to the combined approach based on the conditions below).[10]

In either the separate or combined approaches, there will be an overall statement with a point estimate and confidence interval for all sampled strata taken as a whole. The separate approach will provide a different overall point estimate and precision computation when compared to the combined approach. In addition, in separate estimation, the point estimates are additive when coming up with an overall point estimate. This is not true of the combined approach.

If requested by the taxpayer, either separate ratio or separate regression estimator can be used as the final estimator to project error if:

- It is agreed to be part of the sample plan prior to selecting the sample,

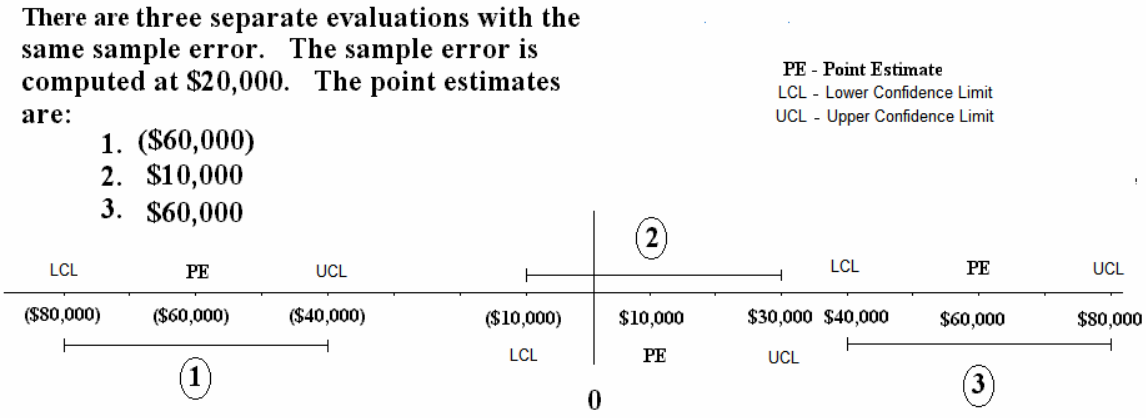
- It will provide for the better precision when compared to other estimators (including the combined estimators), and
- The restrictions shown in section 0308.20, when applied to the strata individually, are satisfied.

0308.40 Evaluating The Estimators

For each estimator, a precision amount can be calculated using the standard formulas found in Appendix G. By adding and subtracting the precision amount to the point estimate (PE), a confidence interval is computed (the width of the confidence interval is always twice the precision amount). The confidence interval is bounded by the upper confidence limit (UCL) and lower confidence limit (LCL). [11]

Confusion exists over what represents the UCL. The procedure for arriving at the UCL and LCL is the same for both positive and negative point estimates, and is shown as follows:

Exhibit 3.1



An auditor can recommend an adjustment at the PE, LCL, or UCL. The PE represents the most likely adjustment (for example #1 of Exhibit 3.1 this is negative $60,000 and for example #3 this is a positive $60,000). The confidence limit closest to zero represents the least expected amount

of the adjustment at a given confidence level (for example #1 the UCL of negative $40,000 and for example #3 the LCL of positive $40,000). The confidence limit farthest from zero represents the greatest expected amount of the adjustment at a given confidence level (for example #1 the LCL of negative $80,000 and for example #3 the UCL of positive $80,000).

Example #2 of Exhibit 3.1 is an example where the possibility of both refund and assessment positions are contained within the confidence interval. Notice that the LCL is a negative $10,000 and the UCL is a positive $30,000. Consequently, the sample results cannot be used to recommend an adjustment based on a projected result. In that event, the only options available are adjusting the sample items without projection or selecting an entirely new and larger sample. Note that a larger initial sample size may have provided a confidence interval where the UCL and LCL are of the same sign.

0308.50 Determining the Optimal Estimator

Statistically, the narrower the confidence interval, the more reliable the point estimate. Therefore, the Commission's policy is to use the estimator that provides the smallest precision amount as the basis for projection. The relative precision of an estimator will not be used as a basis of deciding amongst the estimators (however, in order to support an adjustment, the estimator must provide sufficient relative precision as set out in section 0308.70).

Also the UCL and LCL of the confidence interval must have the same mathematical sign to support an assessment (examples #1 and #3 in exhibit 3.1). If the signs are different, that is the LCL is negative and the UCL is positive, the sample results indicate both a refund and a tax

assessment are reasonably possible. Therefore no adjustment using the sample results will be recommended. If the UCL and LCL from any of the estimators are of different signs, this does not preclude the use of another estimator where the mathematical signs of the UCL and LCL agree.

0308.60 Confidence Level

Samples will be evaluated using at least the 90% confidence level (two-sided). If the taxpayer requests a confidence level higher than 90%, this should be addressed before the sample size is determined.

0308.70 Relative Precision

The goal in setting sample size is to achieve a relative precision of 30% or better. If this is not attained, the sample results may still be accepted provided that both the Commission and the taxpayer agree to go forward and project the results despite the fact that the achieved relative precision exceeds the desired precision.

In the event that desired relative precision is not attained, and the taxpayer is not in agreement with projecting a sample result, than the Commission must be provided an opportunity to sample (or audit) further. The Commission and the taxpayer reserves the right to sample or audit further if the desired degree of relative precision is not achieved.

0308.80 Minimum Number of Errors

Each of the estimation methods described above: mean-per-unit, difference, ratio, and regression - will provide a confidence interval around the point estimate. Creation of a reliable confidence interval assumes that the underlying sampling distribution for that estimator is

approximately normal. Where the underlying sampling distribution is not normal, the UCL and LCL may not be consistent with the expected probabilities. For example, given a two-sided 90% confidence level, the risk that the true unknown is less than the LCL is about 5% given a normally distributed sampling distribution. Similarly, there is about a 5% risk that the true unknown amount is greater than the UCL. Given a non-normal or materially skewed sampling distribution, the true risks in this example for the LCL and UCL might be materially different than the expected 5%. Further, it is known that as the proportion of the sampling units with a zero value increases in the population, the more likely that at any given sample size, that the underlying sampling distribution is not normally distributed. To counter this, larger sample sizes are needed for populations with a large proportion of zero values. Instituting a standard whereby a minimum number of non-zero error amounts are required will aid in the effort of achieving a large sample.

To observe the required minimums often calls for very large sample sizes. This is for several reasons. In Commission audits, most of the concern is over the total amount of the taxable error. Samples are usually taken from populations that contain many non-errors (zeroes) and a few errors (non-zeroes). The occurrence rate of errors could be smaller than 1% some of the time. Many zeroes in the sample may cause problems, particularly with the assumption that the underlying sampling distribution is normally distributed as explained above.

Therefore, in Commission audits, at least three non-zeroes must be observed in any stratum that is included in a statistical evaluation. Note that this policy pertains only to probability samples that are statistically evaluated. Also, where at least one stratum contains three or more non-zeros, but one or more do not have at least three, that the strata

exceeding the minimum can be evaluated without the stratum/strata satisfying the requirement.

0308.90 Taxpayer Disagreement with Results

In the event that the taxpayer disagrees with a finding based on a statistical evaluation, the CAS should discuss the issues with the auditor and his/her supervisor and Audit Director. A meeting with the taxpayer may be needed. Determination should be made on what areas the taxpayer disagrees with. If any deviations from the sample plan have occurred, these need to be addressed or corrected.

If there is concern about precision, Commission and the taxpayer can agree to increase the sample size over all strata or within a stratum. If the taxpayer wishes to expand the sample results that otherwise satisfy the relative precision requirement (rather than re-sampling), then the Commission will not agree to enlarging an existing sample unless the taxpayer commits to agreeing to the results of the increased sample size, before the results are audited. If it is agreed that the results of a prior sample is increased - rather than taking an entirely new larger sample - no portion of the sample will be ignored or replaced. If the taxpayer requests expansion of the sample, or an entirely new sample, the taxpayer is responsible for retrieving the additional documents for review.

0308.95 Adjusting the Sample after the Fact

Removing any single item from the sample may not be done. However it is permissible to identify and remove from the target population a class of items. This then requires that the qualifying class items in both the sampling frame and sample be removed as well. Therefore, if items

identified to be of a certain class are removed from the sample, then they should also be removed from the sampling frame along with other sampling units not selected into the sample. Examples of items that can be considered a class include things like shop supplies, assets, divisions, and transactions made in certain locations.[12]

0309.00 Documentation Requirements

The following details minimum documentation requirements for statistical samples:

- Maintain a description of the sampling plan (refer to Appendix A)
- Maintain a description of the sampling frame. This description should include details on counts and amounts that have been removed from the data download to arrive at the sampling frame.
- Identification of the source of random numbers. The seed numbers provided by the random number generator should always be documented. Any reports made by the program to document the seed should be retained.
- Include instructions to the parties involved in reviewing the sampling units. The auditor should be provided instructions on basic procedures concerning the sample and electronic files (refer to Appendix C).
- Include work papers that contain descriptions that support the audit conclusion for each sample item. A zero or non-zero error amount should be established for each sample item. A non-error sample item should not be deleted from the files. Original sample amounts and other data fields should not be edited.
- Identify and explain special valuation items (refer to section 5).
- Include work papers that contain the evaluation and projection procedures. This should also include a statistical statement of confidence or precision (refer to Appendix G).

- When appropriate, include a statement as to any unusual factors or complications in the sampling process and their implications.

0309.10 Location of the Documentation

The documentation requirements listed above can be found in various reports:

- Computer Audit Specialist's Work papers
- Auditor's Work papers
- Final Audit Report

The following sets the standards as to where the documentation enumerated above is found in a CAA.

1. Computer Audit Specialist

The CAS's work papers should contain:

- Documentation supporting the sampling procedures
- All written communications concerning high or low dollar cut off, items of interest, number of strata, or sample related issues.
- The sampling plan (Appendix A) and instructions to the parties involved in reviewing the sampling units.
- A description of the sampling frame.
- The seed numbers.
- The work papers that contain the evaluation process and projection procedures Appendix G).
- Statements describing any unusual factors or complications in the sampling process and their implications.

2. Auditor

The auditor's work papers should contain:

- All sample items along with descriptions, conclusions and error valuations for each.
- Explanations of special valuation items.
- The work papers that contain the evaluation process and projection procedures (Appendix G).
- Statements describing any unusual factors or complications in the sampling process and their implications.

3. Audit Report

The audit report should contain:

- The sampling plan (Appendix A) and instructions to the parties involved in reviewing the sampling units.
- A description of the sampling frame that identifies included and excluded items from the download.
- All sample items along with descriptions, conclusions, and error valuations for each.
- Work papers that contain the overall statistical evaluation.
- Schedules providing the tax estimate based on the statistical evaluation.
- The narrative to the taxpayer explaining the statistical sample.

PROCEDURES APPLIED TO SAMPLES

0401.00 Non-statistical Sampling and Judgmental Samples

If it is not practical to apply probability theory to evaluate the audited results from a probability sample, the auditor can elect to use non-statistical procedures, provided non-statistical methods can be justified under the three standards expressed in section 0104.00. Sections 0401.00 through 0410.00 apply when statistical procedures are not applied to samples.

Although sections 0401.00 through 0410.00 generally apply to non-statistical sampling procedures applied to probability samples, these concepts can be generally applied to judgmental samples.

Non-statistical sampling techniques will generally be used when the taxpayer is ***unable*** to provide electronic records of the population under examination. Due to the size of most taxpayers that the Commission audits, the auditor should initially make the assumption that the taxpayer will be able to provide electronic records. Therefore use of non-statistical procedures warrant justification within the audit report.

The Commission policy is that the assigned Computer Audit Specialist (CAS), and ***not*** the auditor, determines whether the taxpayer is able to provide electronic records. The auditor is required on all new audit assignments and before the audit commences to contact the CAS. The auditor should contact the CAS as early as possible so that the CAS and auditor can coordinate their efforts. The auditor will be responsible for providing the CAS with the following information on all new assignments:

- Name of taxpayer
- Location of taxpayer
- Name of taxpayer contact

- Contacts telephone number
- Date of assignment
- If already made, date of first audit appointment

0402.00 Defining the Audit Objective

Before any attempt is made to conduct any type of sample, the auditor should clearly define the audit objective(s). In some instances the auditor will have more than one objective. The auditor's objective should be discussed with the taxpayer and be included as part of the sampling plan.

Defining the audit objective serves several purposes. It provides the taxpayer a clear understanding of the auditor's intentions. Defining the audit objective will also assist the auditor in determining the appropriate population(s) to examine. Clarifying the audit objective will assist the auditor in determining what areas might be sampled or reviewed in detail.

0403.00 Define and Refine the Population

The auditor should give careful thought to the population to be examined. By refining the population, the better the chance that the auditor will get useable sample results. In the long-run, the more effort that is made by the Commission auditor in refining the population, the more accurate the results will be. Proper planning at this stage is also a key to auditor efficiency.

Having a stated audit objective will greatly aid the auditor in defining the population. To the extent that it is possible, the auditor should refine the population so that it matches the population of interest. When records are not available electronically, refining the population is often difficult. However, there are still a number of ways in which the population may be refined.

0404.00 Sampling Unit, Sampling Frame, and Items of Interest

Sampling units are the individual elements of the population/sampling frame that will be audited if selected into the sample. For example, if the population consists of sales, then the sampling unit could be each sales invoice. The sampling frame here will be the list of all sales invoices. Another example might be a sampling unit consisting of transactions grouped together for a time period. Suppose the auditor has an audit period for a 36 month period. All transactions or invoices, either sales or expenses, are grouped (or clustered) together in each month. The auditor will sample the months as sample units. If selected, all invoices for a selected month are audited. How the auditor defines sampling units is generally a function of what is available.

In some instances there may be more than one choice in how the auditor defines the sampling unit. The goal is to obtain the sampling unit that provides the least variability between all sampling units in the population that is of audit interest. In cases where more than one sampling unit is available a determination will need to be made as to which offers the auditor the greatest efficiency. Efficiency as defined here is meant to be a comparison of cost associated with locating each sampling unit, the expected return, and desired accuracy. Even though a sampling unit may yield a higher degree of accuracy in the final results of the sample, it may not always be the best choice. Likewise, the most convenient sampling frame may provide too imprecise of a sample result.

The following is the Commission's preference of sampling units. These are meant to be general guidelines in helping the auditor to decide which sampling unit should be used:

(1) Electronic records…..line items then invoice level

(2) Transactions (e.g., invoices, checks, purchase orders)

(3) Clusters

(4) Time periods

(A) Days/Batches

(B) Weeks

(C) Months

(D) Years

Sampling Units (1) through (4) can be chosen into the sample using either probability or judgmental sampling methods. However, when using sampling units (1) through (4) as a basis for an audit finding, probability sampling should be used if possible. Note that if the auditor has electronic records (1), it is expected that use of statistical methods should be applied as described in sections 0301.00 through 0309.00. If statistical methods are not applied, then there is an expectation that the auditor explain why non-statistical methods were used.

With regard to a "transactional" sampling unit (2), statistical or non-statistical methods may be applied if the sample is a probability sample. Often, the reason non-statistical methods are applied is that it is known before a sample is even taken that an impractically large sample needs to be taken to otherwise apply statistical methods. In any case, this choice should be explained by the auditor in the audit report.

The auditor may apply statistical methods to cluster samples (3) that qualify as a probability sample. However, it is more common to apply non-statistical procedures to a cluster sample.

With regard to time period samples (4) (actually a form of cluster sampling), it is usually not appropriate to apply statistical evaluations to these type of samples; even if these samples qualify as probability samples. Note that if the auditor elects to use a time period sample, that days are preferred over weeks, weeks over months, and months over years.

Finally, the collection of all sampling units that has a chance of being selected through probability sampling methods, no matter how defined (1) – (4) above, makes up the sampling frame. The sampling frame constructed by the auditor (as described in section 0403.00) should include any transaction or part of a population recorded in the accounting records during the period examined that:

(A) Has a likely impact to the amount of reported tax for sales and use tax purposes, and

(B) Is under the scope of the audit objective.

The criteria (A) and (B) describe ***items of interest.*** For example, sales of tangible property would likely be of interest, while sales of company stock will likely not be. The auditor is to use judgment in determining what is likely of interest, and make best efforts to define a sampling frame that includes items of interest, and further refine the frame to exclude items not of interest, if logistically and practically possible. The greater the percentage of the frame containing items not of interest, the larger the sample has to be for both statistical and non-statistical methods to provide a sufficient sample upon which to base an audit finding. Not clearly understanding or stating the audit objective before sampling, or if items of interest are not in the frame, the auditor likely will have to perform a separate examination for those items, leading to inefficiencies.

According to the disclosure standard in section 0103.00 the auditor is obligated to describe to the taxpayer what generally constitutes an item of interest prior to sampling. The auditor should consider any reasonable request made by the taxpayer in modifying or expanding the scope of what is defined as an item of interest, and report any objections in the final audit report if no resolution on the issue was reached.

Although it is preferred that the auditor remove any items not of interest from the sampling frame, it is not necessary to remove all items not of interest. A sampling frame that contains items not of interest is still valid, but will likely require larger samples to attain a more accurate result when compared to a frame that has these items removed. As such, under the BEST EVIDENCE standard, the auditor is obligated to make a reasonable attempt, given the time, information, and resources available, to remove items not of interest from the sampling frame.

0405.00 Sampling Method

The primary reason for probability sampling in non-statistical sampling is to provide an unbiased selection from across the sampling frame (note that the sampling frame is created using auditor judgment). Eliminating any bias in the selection process is of utmost concern to the Commission. By using probability sampling, selection bias (known or unknown) - that of the auditor or taxpayer – can be eliminated.

Probability sampling will also lead to more precise results over judgmental sampling where sample size, in terms of percentage of the population reviewed in the sample, is approximately equal between these two basic forms. In many cases, probability samples, often of significantly smaller proportions in terms of the percentage of the population selected into the sample, will also lead to similar or even greater accuracy when compared to judgmental samples. Probability sampling also increases the defensibility of the sample. While the auditor and taxpayer may prefer a monthly sample to be chosen non-randomly (using judgment) for the sake of convenience, random selection of the months should be used.

Probability samples, including simple random samples, should be taken only through the use of software sanctioned by the Commission. If the auditor

intends to base any audit finding from a probability sample drawn using any other means, including samples drawn by the taxpayer or their representative, this must be approved by the CAS administrator.

If the records are available electronically, the auditor should use stratified random sampling rather than a simple random sample (unless the population is too small to justify a stratified random sample – then a simple random sample can be considered). Where the population is not available electronically, stratification is generally not practical, then the auditor should consider a simple random sample, provided that the auditor can establish correspondence that allows matching of random numbers to sample units. Where correspondence is difficult or impractical, the auditor should then consider judgmental sampling. Before using judgmental sampling techniques, the auditor should consult with a CAS – possibly some efficient means of establishing correspondence was overlooked.

0406.00 Sample Sizes

Choosing the appropriate sample size is always of critical importance even in a non-statistical sample. One rule to remember about sample size is that more is generally always better when considering accuracy. However, this does not mean that we want or need larger sample sizes than what are needed to obtain the desired results. Choosing the appropriate sample size is always a trade-off between auditor efficiency and precision.

The Commission auditor should use the following guidelines when calculating sample sizes for **non-statistical** samples.

0406.10 Transaction Sampling – Sample Size

The minimum sample size for any sample when the transaction is the sampling unit is a sample size that has a likely probability (at least 50%

chance) of uncovering at least 250 **items of interest**. This does not mean that sample size, that is the number of sampling units contained in the sample, will always be 250. In most cases, the minimum sample size will be more than 250. The auditor may not draw smaller sample sizes less than this limit without the approval of the CAS Administrator. Note that if the sample has materially less than 250 items of interest in the sample, and this outcome was known to be likely before sampling, the auditor did not comply with the minimum sample size requirement of this section.

Before drawing the sample the auditor should discuss with the taxpayer the proposed sample size to determine if the taxpayer has information that would indicate that another sample size would be more appropriate considering the desired results. The Commission reserves the right to increase the size of the sample if the sample is deemed insufficient upon which to base a satisfactory, reasonable, and proper audit finding. Note that such a finding may require a sufficiently large sample to assure that significant error does not exist in the population. Before increasing the size of the sample the matter should be discussed with the taxpayer and the auditor's supervisor. The taxpayer also has the right to request an increase in sample size. Should the taxpayer request an increase in sample size they will be obligated to provide the auditor with the additional sampling units for review by the auditor.

The formula for computing non-statistical sample sizes is:

$$Minimum\,Sample\,Size = \frac{Items\,of\ Interest}{\%\,of\ Items\,of\ Interest\,in\,Population}$$

0406.20 Cluster Sampling – Sample Size

Cluster sampling is used for efficiency reasons. There is a trade-off between efficiency and precision. Some precision will be sacrificed for

efficiency reasons when using cluster sampling. The minimum number of clusters to use by the auditor when using cluster sampling is 100. The number of elements in the cluster (cluster width) must be sufficient so that at least 250 items of interest are likely to be uncovered in the sample.

It should be remembered that when using cluster sampling the cluster becomes the sampling unit and not the components that make up the cluster (often an invoice or line amount). The auditor will still audit the same number of transactions that was computed in the sample size but it is the cluster that will be evaluated and not the transactions.

0406.30 Time Period Sampling

Time period sampling is ***not*** the preferred method of sampling and should only be used when it is not possible to use the transaction as the sampling unit. When time period sampling is used the auditor will obtain approval from his/her supervisor first. The auditor will also include in the narrative of the audit the reasons why transaction sampling could not be used.

Although selection of the time period sampling units should be made using random methods, the samples themselves should never be considered satisfactory so that statistical sampling procedures can be used to appraise the results.

0406.40 Day as the Sampling Unit – Sample Size

When using day sampling as the sampling unit the ***minimum*** number of days that will be examined will be 30.

0406.50 Week as the Sampling Unit – Sample Size

When using the week as the sampling unit the ***minimum*** number of weeks that will be randomly selected and examined will be 20.

0406.60 Month as the Sampling Unit – Sample Size

It is the policy of the Commission that when using the month as the sampling unit the auditor will randomly select 1-month for each 12-months under audit. For example, most states, the statute of limitations is 36 months. For those states with a 36-month statute of limitation the number of months that will be audited and selected by the auditor will be 3. For those states with a 48-month statute of limitation the number of months to be selected will be 4. For a state with a statute of limitation of 60-months the number of months to be audited will be 5.

0407.00 Systematic Sampling

Under some circumstances, the most convenient method of selecting the sampling units could be the use of systematic selection techniques. If done using random procedures, a systematic sample can be considered a probability sample. Generally systematic sampling will be considered when it is difficult or impossible to match random numbers with sampling units (correspondence).

If conducted properly, a systematic sample can provide sufficient evidence such that a satisfactory, reasonable, and proper audit finding can be made. This is possible since the sample items are spread evenly across the population.

Unlike a simple random sample, a systematic sample has a weakness in that it is much more susceptible to problems if any continuous and repetitive patterns exist in the population. Should the systematic sample pick up a pattern in the population, it may cause the sample to be deemed insufficient for providing a

satisfactory, reasonable, and proper audit finding. To avoid this, the Commission policy is that the auditor will use multiple starting points.

The auditor should use good judgment in determining the number of starting points to use. The ***minimum*** number of starting points for any systematic sample will be **3.** Before attempting a systematic sample, the auditor should be familiar with and follow the steps as in explained in the MTC sampling courses. If needed, the auditor should seek assistance from a CAS.

0408.00 Sufficiency of a Non-statistical Sample

The aim of the auditor is to substitute partial coverage with total coverage without material dissimilarity between the overall results of either method. Without applying statistical sampling evaluation procedures, it is not possible, using any objective method, to obtain an audit finding that accurately measures, with some reasonable degree of confidence, whether the total error projected from a probability sample estimates the true and unknown total error. However, a sample result may show, without using statistical evaluation procedures that:

- Errors have been made by the taxpayer in the **selected** sample units,
- It can be reasonably be expected, using auditor judgment, that more sample units in the sampling frame, having a chance of being selected but were **not selected** into the sample, will be in error, and
- It can be expected, again using judgment, that such errors not sampled are of the same or similar nature to those in the sample at approximately the same rate of occurrence and/or proportionate value.

If the nature of the errors found in the sample allows the auditor to conclude other un-audited errors still exist in the population, the auditor should consider coming to an audit finding that recommends an adjustment or change to the taxpayer reporting. This adjustment amount can be made based on

further auditing of additional sampling units, or from the projected results from the audited sample using methods described in section 0410.00.

Fundamental to determining the sufficiency of the sample, the auditor must determine:

(i) Is the sample itself adequate in size to determine whether a satisfactory, reasonable, and proper audit finding can be made?

(ii) If the audited sample results contain tax reporting errors, does the nature of the errors indicate that more errors exist in the un-audited sampling units?

(iii) Is it reasonable to presume that the frequency and amount of the errors found in the sample occur at about the same rate as all the other un-audited sampling units?

If the auditor can answer "yes" to all three of these questions, then the auditor should project (extrapolate) the sample results to the entire population that had a chance of being selected into the sample.

0409.00 Judging the Nature of Errors Found in the Sample

Using auditor judgment is really all that can be done using non-statistical methods. The best that the auditor can do is decide on the sufficiency of the errors by answering the basic questions (ii) and (iii) stated in Section 0408.00. Further, a reasonableness test can be applied to a non-statistical approach: can it be reasonably expected that an independent and disinterested third party to the audit agree with the auditor's conclusions with regard to (ii) and (iii) of Section 0408.00? If the taxpayer disagrees with the auditor's conclusions with respect to (ii) and (iii), the auditor should consider expanding the sample. If the expanded sample bears out much the same result, then the auditor should consider projecting (extrapolating) the sample under the provisions of section 0410.00.

0410.00 Projecting a Non-statistical Sample

There are many methods of projecting sample results. In non-statistical sampling, the ratio and difference estimators are the most commonly used. Of the two estimators, the ratio estimator will normally be used. The difference estimator is used less frequently because a count of the sample and population items is needed. In cases where the population is not electronically available to the auditor, the counts are usually not available or are difficult to obtain.

In situations where either estimator can be used, the auditor will discuss with the taxpayer and come to an agreement as to which will be used before the sample is drawn. Before using any other method of projection the auditor will discuss with and obtain approval from his/her supervisor.

SPECIAL AUDITING CONCERNS AS THEY RELATE TO SAMPLING

0501.00 Valuing the Sample

If a probability sample is to be projected and included in an audit assessment, the auditor is required to come to an audit conclusion, that is, an error amount, on every sampling unit drawn in the sample. The error amount may be a zero or non-zero amount. The valuation procedures should be, in most instances, the same as if the item was examined in a detailed (actual) audit. However, some sample units drawn in probability samples may require special valuation procedures. This section addresses some of these special circumstances.

0502.00 Missing Items

A missing item is a source document representing a sampling unit that has been drawn as a sample item and the supporting documentation cannot be located or was not provided to the auditor. Missing invoices constitute a possible attribute or characteristic of an accounting population. According to Arkin,[13] a probability sample is bound to have missing items if it is drawn from a population that has many missing documents. He further states that:

> *It is not sufficient in the case of a document missing from the [population] for an auditor to take another sample item in its place without running down the missing document, unless the auditor is willing to restrict his/her conclusions only to those items remaining in the file to recognize the deficiency.*

If the source document cannot be located in the taxpayer's records and is otherwise not available; there are three basic options in valuing the sample unit. They are:

1. The item may be accepted as reported with no adjustment based on auditor's judgment

For example, a missing item represented a common purchase made from a vendor that always charged tax on similar transactions in the audit period.

2. A partial adjustment may be made based on alternative evidence or procedures

   For example, a missing invoice from a vendor with a history of errors was included in the sample. The auditor used the proportionate errors found in the other transactions and applied it to the missing invoice.

3. The item may be considered unsubstantiated and totally adjusted

   For example, a transaction represented the only purchase from a vendor. The auditor treated the purchase as an error.

Treating a missing item as totally in error is the last resort. This option should be used if the taxpayer is unable to provide sufficient evidence regarding the missing item.

Individual missing items should never be replaced in a sample. Replacing the sample containing the missing items with an entirely new sample is an option. However, if there were a significant number of missing items in the first sample, it is likely that another sample will also contain missing items.

Although the treatment of missing items in financial auditing is not dispositive to sales and use tax auditing, it is noteworthy to observe how financial audit standards relate to this issue. The AICPA published an *Audit Sampling Guide* that discusses random sampling for test of controls and compliance testing and makes the following recommendation on page 31:

> *If that document cannot be located or if for any other reason the auditor is unable to examine the selected item, he or she will probably be unable to use alternative procedures to test whether that control was applied as prescribed. If the auditor is unable to apply the planned audit procedures or appropriate alternative procedures to selected items, he or she should consider selected items to be deviations from the controls for the purpose of evaluating the sample.*

Procedures used in tax audits are similar to financial audits with the exception that the item will not automatically be considered an error.[14]

0503.00 Extraordinary Items

The auditor is required to discuss and attempt to reach agreement with the taxpayer on the sample frame and sampling unit. After agreement has been reached (or if all reasonable requests from the taxpayer have been considered), a probability sample is selected from the sample frame.

All items within the probability sample are included in the sample frame and are not 'extraordinary items' separate and apart from the sample frame.

No adjustment should be made to delete items from the probability sample selected. If the taxpayer is not in agreement, the auditor should discuss this issue with the computer audit specialist.

0504.00 Corrections and Reclassifications

It is acceptable to consider evidence from outside the sample in determining the value of error for the sample unit selected. The transaction, related documents, and accounting entries should be followed to their logical conclusion. Examples include:

- A reclassification of an expense item to another account included within the sampling frame. All entries to both accounts are sampling units within the sampling frame, but only one was selected in the sample.
- A reclassification of an expense item within the sampling frame to an account outside the sampling frame.

In the first example, the transaction had more than one opportunity to be valued for error because the original and correcting entries were both in the

sampling frame. If the transaction is in error, this has a potential to bias the sample results, unless a decision rule is followed:

If any entry other than the final entry is selected in the sample, it can never be valued as an error other than $0. Only the final accounting entry, if selected in the sample, should be valued for taxable error.

The following is another example:

> The first invoice is $80, the second invoice is a credit memo for $80, and the third invoice is the corrected invoice in the amount of $75. All invoices are in the sampling frame. The transaction is in error. If the first invoice or second invoice is selected in the sample, the sample item should be valued as a $0 error. The sample item would be valued as a $75 error only if the third invoice is selected in the sample.

The auditor needs to explain to the taxpayer that it is the taxpayer's responsibility to provide evidence that the transaction has been adjusted.

0505.00 Multiple Items

A "multiple item" is one sampling unit that is a collection of more than one transaction, where one transaction is expected. Often, sampling units are summaries of many transactions. Here are some examples of multiple items:

- Credit card entries
- Employee's expense reimbursement
- Petty cash reimbursement
- Checks

All components of the sampling unit should be valued for error.

0506.00 Installments

If the sample unit selected is part of an installment transaction, then the entire transaction should be examined. If an error is found, then it should be

projected to the sample item proportionate to the entire transaction amount.

The following is an example:

> A transaction is billed on four invoices. Each billing is for $1,250 for a total of $5,000. One of the invoices for $1,250 is selected in the sample. Three more installment payments for $1,250 exist but were not selected. One of the four invoices is not in the sampling frame. Upon review of the entire transaction, the following is determined:
>
> Of the $5,000 total transaction, $2,000 is in error.
>
> The formula to value the error is:
>
> $$\frac{\text{Total Error}}{\text{Total Transaction Amount}} \times \text{Sample Unit Amount}$$
>
> $$\frac{\$2{,}000}{\$5{,}000} \times \$1{,}250 = \$500$$
>
> The valuation for the sample unit error is $500.

APPENDIX

A

SAMPLING PLAN

Sampling Plan and Documentation

Taxpayer Name		**FEIN**	
		State ID	
Audit Period		**Auditor-in-Charge**	
	To		
State			

This form should be included with the audit file as documentation for all sampling procedures and all decisions made by the auditor concerning sampling.

Sample Plan Documentation	*Yes*	*No*
Has the sampling plan, along with any modifications, been discussed with the taxpayer?		
Has a sampling plan letter been provided to the taxpayer? *(if yes, please attach)*		
Were there any significant modifications to the sample plan after the sample was drawn? *(If yes, please describe below, or otherwise provide documentation, along with any discussions with the taxpayer regarding changes)*		

Audit Objective – Reason to Sample

Target Population Description *(for example: sales, purchases, fixed assets, or expenses for manufacturing division)*

Sampling Frame Description
Generally describe the sampling frame below, and attach any documentation summarizing the frame. Include the period(s) for which an electronic download was received if it differs from the audit period described above. *(e.g., electronic download of purchase orders, general ledger entries, or list of claimed deductions)*
Describe the sampling unit. *(the individual elements of the sampling frame that will be drawn into the sample, e.g., download line items, clusters of invoices, or all transactions in randomly selected time periods)*

Describe any refinements made to the sampling frame that will not be sampled as they likely will not meet the audit objective, are not under the scope of the audit, or possibly will be handled separately in a detail or another sample.

(items removed, such as inventory, sales to certain vendors, purchases made in divisions not located in the state or from certain accounts)

Frame and Sample Summary

Stratum	Stratum Description	Total # of Sampling Units	Total Invoice Value of Sampling Units	Sampling Units Selected into Sample or Audited 100%	Seed Number
	Total Sampling Frame				

Sampling Procedures *(please fill out the table labeled below as "Frame and Sample Summary")*	*Indicate with an "X"*
Indicate the sample design used: *(please select only one from the following)*	
Stratified random	
Simple random	
Cluster	
Time period (random selection)	
Time period (judgmental selection)	
Judgmental block (describe below)	
Other probability sampling (describe below)	
Other judgmental method (describe below)	
Description of judgmental or "other" sample designs used *(if applicable)*:	

	Indicate with an "X"
Indicate the software used to draw the sample or provide the random numbers: *(please select only one from the following)*	
Multistate Tax Commission Sampling Software (MSS)	
Other software (please describe below)	
Other random method (please describe below)	
Taxpayer drew/provided random numbers (please describe below)	
Description of "other" source of random numbers *(if applicable)*:	

	Indicate with an "X"
Indicate how sample size was determined: *(please select only one from the following)*	
Software recommendation using expected error rate (describe error rate)	
Using the suggested minimums	
Negotiated with the taxpayer (describe issues that relate to sample size)	
Other method (describe below)	
Comments concerning sample size issues:	

Sample Valuation Issues	*Indicate with an "X*
Indicate the value estimated in the sample: *(please select only one from the following)*	
Taxable Error	
Tax Error	
Taxable Amount	
Tax Amount	
Other (please explain below)	
Other variable being valued in the sample:	

	Yes	*No*
Were any sample units removed from the sample (not valued)? *(If, yes please explain below)*		

	Indicate with an "X
Indicate if any special valuation issues below were encountered, and explain these issues below: *(please indicate any that might apply)*	
Missing Items (sample units lacking documentation)	
Sample unit where documentation/records outside the sample frame were considered in valuation	
Installments	
Duplicated items	
Voided transactions	
Other valuation issues (please explain below)	

Describe any special valuation issues encountered in auditing the sample:

Sample Evaluation and Projection	*Yes*	*No*
Was the sample statistically evaluated? *[If "Yes", please attach work paper showing evaluation using MTC Sampling Software (MSS),* *Otherwise, if "No", please explain why no evaluation was performed]*		
If the sample was statistically evaluated, was the taxpayer provided the results of the evaluation?		
Will the audit report recommend an adjustment based on a sample projection (whether or not a statistical evaluation was done)?		
If the audit report includes a projection and the sample was statistically evaluated, does the statistically recommended value from MSS agree with the projected value used in the audit report upon which a tax adjustment is made? *(If "No", please explain)*		
If the audit report will include a projection, was the projected value allocated between time periods (tax rate/penalty/interest considerations) and/or locations (local tax rate considerations)? *(If "Yes", please explain and attach work papers showing allocations)*		

APPENDIX

B

SAMPLING LETTER

Date:

Address:

The following explains the proposed sampling procedures to be used by the Multistate Tax Commission in the audit of **[INSERT TAXPAYERS NAME]**. Attached is a document that includes the details to the sampling plan.

The area of the audit examination that would utilize Computer Assisted Audit Techniques involves the review of **[INSERT WHETHER SALE/USE OR BOTH**] within the listed states and for the following period(s) (See attached list of participating states and audit periods). The **[INSERT WHETHER SALE/USE OR BOTH**] would be reviewed for possible overpayments and underpayments of sales tax and/or use tax. The following briefly summarizes the steps in the process.

1. The first step is to identify the necessary data fields required in the electronic data. Typically such fields include reference date (invoice, purchase order, etc), reference number, item description, general ledger code, purchase amount, vendor number, vendor name, retail sales tax charged by the vendor (if available), and use tax accrued (if available).
2. Next, the electronic data by account number will be summarized. This summary will be provided to the auditor so that the "accounts of interest" can be identified. Accounts with little or no possibility for errors in the payment of sales or use tax may be eliminated. Examples of account types, which are candidates for elimination_____.
3. Once agreement has been reached between the auditor and taxpayer on the accounts of interest, these records, without the excluded transactions, will be placed into a separate file for the purpose of sampling. Please note that both the auditor and taxpayer need to identify and agree on the basic "sampling unit". If the sampling unit is the invoice line item, then only the selected line items will be valued for possible tax error – not the entire invoice. On the other hand, if the invoice is chosen as the sampling unit, the entire invoice may be valued for error. This agreement will determine the population (excluding amounts to be reviewed on an actual basis, negative and zero dollar amounts) from which the sample is selected.

4. High dollar transactions will be excluded from the population and examined on an actual basis. A lower level threshold may also be established. The auditor and taxpayer will agree upon these thresholds.
5. All samples will be selected randomly using a random number generator. The random seed will be documented to allow for expansion or recreation of the sample if necessary. Negative and zero dollar amounts will not be normally directly sampled. However, it will be possible to reduce or eliminate a debit or credit error from the sample if a negative item reduces, cancels, or voids the same transaction included in the sample. Also, a separate test of such items can be designed and generated if agreed upon by the auditor and taxpayer.
6. The population (accounts of interest) will be stratified, by amount, into two to six strata plus actual (detailed stratum). This is to reduce the overall variation in the sample. The procedure for determining the exact number of sampled stratum and stratum breaks will be determined mathematically according to a procedure explained in the sampling plan details. Generally, sample size will be approximately 200 - 400 items per strata plus the detailed stratum. In rare instances a simple random sample may be warranted.
7. If there are less than three errors in a stratum, that stratum is ignored in the final evaluation. Any error included in a stratum that is not projected may be subject to tax on an actual basis.
8. The errors found in the strata sampled will be projected to a statistically conservative estimate of the tax due. One of the following estimators will be used; stratified mean-per-unit, stratified difference, combined ratio, or combined regression. The estimator that provides the best precision will be used to project any taxable errors valued in the sample.
9. The projection method will use the 90% confidence level (two-sided). In the event that the upper and lower limits are of different mathematical signs, no projection will be made.

If you have any further questions please do not hesitate to contact me at _____.

Computer Audit Specialist

APPENDIX

C

SAMPLING LETTER ATTACHMENT EXPLAINING SAMPLING PLAN DETAIL

Statistical Sampling Policies

Statistical sampling requires a sampling plan that should be determined before sampling commences. This plan is usually established by state law, various agreements made with the taxpayer, and by agency policy decisions. The Commission and some – but not all – of its member states have developed policies concerning statistical sampling. These policy decisions differ in some areas. In applying policies to statistical sampling in audits, the Commission auditor will first look to the member state's policy for which the audit is being performed. In the absence of a state policy, the Commission's policies expressed herein will be applied.

This document details policy with respect to the decisions or issues that must be made in advance of sampling, and some of the decisions or issues that often may come up as a result of the audit sample. It is difficult to foresee and discuss all possible decisions or policy with respect to every issue within this document. As such, the Commission has adopted a sampling policy manual that will be used for guidance, in addition to this document and any sample plan made with the taxpayer.

Precision and Confidence Level

The goal of the Commission is to obtain 30% relative precision using a two-tailed 90% confidence interval. In the event that the desired relative precision is not achieved, the projected sample results may still be used provided that both the Commission and the taxpayer agree to go forward and project the results despite the fact that relative precision exceeds 30%. In the event that desired relative precision is not attained, and the taxpayer is not in agreement with projecting the sample, than the Commission must be provided an opportunity to sample (or audit) further. The Commission reserves the right to sample or audit further if the desired degree of relative precision is not achieved.

Procedure for Determining Strata Breaks

1. All negative amounts (accounting adjustments) and zero items should be eliminated from the population
2. The total count of the remaining population will be calculated
3. The total dollar value of the population will be calculated.
4. The total count for each defined class will be calculated
5. The total dollar value of each defined class will be calculated
6. The Commission will then define the strata breaks using "Cumulative Square Root of the Frequency". After the strata breaks are determined the file will be returned to you for your review and

agreement. At which time the sample will be drawn from each defined stratum.

Sample Size

Under no circumstances will a sample of less than 100 items be permitted. If a simple random sample is to be used than minimum sample size will be 100. If a stratified random sample is being conducted then the minimum sample size is 100 items per stratum.

There are a number of approaches that can be used to statistically calculate the sample size. In using any of these approaches determining appropriate sample size for the given precision level and confidence interval is a best guess. The standard approach used in Commission audits is to use a sample size of 250 for a simple random sample and 250 – 400 per stratum for stratified random samples. In either case determining the appropriate sample size is an issue that will be discussed between the Commission and the taxpayer, except that Commission will not agree to sample sizes below the stated minimums.

Number of Strata

In any population in which a statistical sample is considered, the population will almost always be divided into several groups or strata, and an independent random samples will be taken from each stratum (a stratified random sample). The primary reason why stratification is preferred is that the accuracy of a stratified random sample is usually better when compared to a sample of the same size taken from an unstratified population (a simple random sample). However, a simple random sample may be taken in cases where the population is too small to justify stratification, or it is known in advance that stratifying will not improve the accuracy of the projected sample results.

The criteria used to stratify on are usually invoice value, book value, or some other amount available from the books and records. In some cases, stratification can be done on other characteristics such as business locations or even a combination of characteristics.

Although stratification will place the population into strata for the purpose of sampling, some of the strata may not be sampled. Sometimes, a low dollar (floor) amount can be established and all records with an invoice value below that amount will be ignored (not audited). Similarly, a high dollar amount (ceiling) is usually established, and all records with an invoice value at or above this amount are not be sampled but audited 100% (detailed). The Commission will make all efforts to

come to some mutually agreeable floor and ceiling amounts with the taxpayer.

The number of strata actually sampled (excluding strata detailed or ignored) will be in most cases three to four strata. In some cases only two strata will be sampled, and other cases up to six strata may be considered for very large populations. Constructing more than six strata will rarely provide any benefit. If the taxpayer has concerns in this area, the issue of the number of strata actually sampled can be discussed, and the Commission will make every effort to come some mutually agreeable number of sampled strata.

Evaluating the Sample

When evaluating statistical samples the Commission uses the following statistical estimators. These estimators can be used in simple or stratified random samples. In stratified sampling, the ratio and regression have two approaches in providing an overall estimate: separate or combined estimation.

1. Mean-per-unit
2. Difference
3. Ratio
4. Regression

The estimator that yields the best precision (that is the smallest precision amount) will be used in making any projection at the midpoint (point estimate).

In evaluating the sample results, a statistical sampling statement can be made that computes a confidence interval that is bounded by the lower confidence limit (LCL) and upper confidence limit (UCL). The point estimate (PE) is always between the LCL and UCL. If the LCL, PE, and UCL of the confidence interval have the same mathematical sign, that is either all positive or all negative, the sample is said to “evaluate” and the estimator can be used to project the sample results. If the signs are different, that is the LCL is negative, the PE is either negative or positive, and the UCL is positive, the sample results are said “not to evaluate”. In the event that an estimator does not evaluate, the point estimate cannot be used to project the sample results. In the event that an estimator does not evaluate, this does not preclude the use of another estimator that does evaluate. Also, in some cases, the Commission will be precluded from using the ratio or regression estimators under some statistical conditions that arise from time to time.

In some cases, all the estimators may not evaluate. In this instance, the Commission will recommend to the states that no projected assessment or refund be made.

If both the Commission and the taxpayer agree, sample size may be increased in an effort to obtain more useful sample results. Should the sample size be increased, then the results of the increased sample can be utilized provided that estimators subsequently evaluate.

Handling of Accounting Adjustments (negatives)

Most accounting populations examined for error will have accounting adjustments, which include things like credit memos, debit memos, reclassifications, or other similar adjustments. Although the sample will be generally taken from a population excluding the accounting adjustments, that does not mean the accounting adjustments will be ignored in the sample. Any applicable accounting adjustments must be considered by the auditor in determination of any error.

First, before the sample is drawn, an examination will be made that matches up accounting adjustments that cancel each other out. These matched accounting entries are then excluded from the population sampled. The remaining population often has unmatched accounting adjustments which will not be sampled. However, after drawing a sample from the remaining items excluding unmatched accounting adjustments, the auditor must consider any corresponding unmatched accounting adjustment to the underlying tax transaction in computation of any taxable error with respect to the sampling units drawn.

In some cases, the accounting adjustment that corresponds to the underlying transaction for any sample unit drawn into the sample is outside the time frame covered by the population or audit period. The auditor should also consider these accounting entries in determination of any error.

Missing Items

A "missing item" is any sample unit drawn into the sample where the taxpayer or its representative, for whatever reason, is not able to provide, or has not provided the necessary source documents and other accounting records that is required by the Commission auditor to make an otherwise satisfactory audit conclusion concerning that unit. In most cases, the required audit conclusion is the taxable (tax) error value of the sample unit. An error value can be zero, negative (tax overpayment), or positive (tax underpayment).

In no case will the Commission consider substituting, replacing, or removing missing items from the sample. Every sample unit drawn into the sample will be valued for error. However, subject to auditor approval, alternative audit evidence may be used in determination of any audit error. In addition, also subject to auditor approval, the audit error may be valued at zero error or only partially in error (based on an examination of similar transactions or other satisfactory evidence). In other cases, the auditor may decide that unless the necessary documentation is provided, the unit will be considered fully in error.

APPENDIX

D

EXPLANATION AND STEPS IN USING CUMULATIVE SQUARE ROOT OF FREQUENCY

Appendix E: Cumulative Square Root of the Frequency (CSFR)

Once the sampling frame is established, there are several ways to determine stratum breaks. In Commission CAA, the "Cumulative Frequency of the Square Root" is the preferred method to establish strata boundaries.

Background

Stratification boundaries can be determined using several approaches. The most common method is CSFR. This methodology has universal acceptance in sampling. There are many references for this methodology in accounting and survey sampling books.[15] The CFSR methodology has been attributed to an article appearing in the Journal of American Statistical Association in 1959.[16]

It would be optimal to establish strata boundaries based on the error amounts. Since the errors are unknown, populations are stratified using the invoice amounts.[17] CSFR will distribute the variation of the invoice amounts in an "optimal" manner to the strata.

CSFR methodology breaks down the population into intervals. These intervals are sometimes referred to as "cells" which can be of equal or unequal width. In most references, the method is described using cells of equal width. When broken into unequal widths, an extra step is involved.[18] The Commission generally uses cells of unequal width because the stratum breaks become more exact. Note that CSFR does not dictate the number of strata, only the breaks between the strata.

Mechanics

The following is an explanation of the procedures used to determine stratum breaks assuming cells of unequal width.

Step 1 Evaluate the population and determine if transactions below a certain dollar level should not be sampled based on the dollar significance of those transactions.

In this example, the taxpayer and the auditor agreed on the accounts of interest and the sample population. No transactions equal to or less than $100.00 will be reviewed.

Step 2 Evaluate the population and determine the dollar amounts that will be reviewed on an actual basis.

All items greater than $25,000.00 will be reviewed in detail (actual basis).

Step 3 Group the remaining transactions within the population of items to be sampled ($100.01 - $25,000.00) into a large number of dollar ranges.

Step 4 Determine the frequency for each dollar range. This is the number of transactions within each range.

Step 5 Calculate the square root of the frequency (number of transactions) for the first range ($100.01 – $150.00). The corresponding frequency was 16,853 and so this value is 129.82. Then calculate the square root for the next range ($150.01 – $200.00). If this frequency is 10,203 then this value equals 101.01. Continue this process for each of the ranges.

Step 6 Calculate the square root of the range width for the first range ($150.01 – $200.00). This value is 7.07. Then calculate the square root of the next range ($150.01 – $200.00). Again, this value equals 7.07. Continue this process for each of the ranges.

Step 7 For each range; multiply the square root of the frequency (step 5) times the square root of the range (step 6).

Step 8 Accumulate the values (as calculated in step 7) for each range. For the first range, the cumulative square root value is 917.83. The cumulative value for the second range is

1,632.97 (714.14 + 917.83 - accumulated value for the preceding range).

For each range, add the square root value (step 7) to the preceding cumulative square root value (step 8) to arrive at the next cumulative square root value. Continue this process for each of the ranges.

Step 9 Divide the total cumulative square root value (say 18,422.66) by the number of strata (4 in this audit) to arrive at cumulative square root value for each stratum. In this audit the value is 4,605.67.

Step 10 Multiply the result of step 9 by 1 to calculate the first stratum range, or 4,605.67. Locate the cumulative square root value closest to this amount. In this case, it is 4,623.62. Locate the ending range associated with this value ($500.01 - $550.00). The first stratum range (boundary) is $100.01 – $550.00.

Step 11 Multiply the result of step 9 (4,605.67) by 2 to calculate the second stratum range, or 9,211.34. Locate the cumulative square root value closest to this amount. In this case, it is 9,213.44. Locate the ending range associated with this value ($2,100.01 – $2,200.00). The second stratum range (boundary) is $550.01 – $2,200.00.

Step 12 Multiply the result of step 9 (4,605.67) by 3 to calculate the third stratum range, or 13,817.07. Locate the cumulative square root value closest to this amount. In this case, it is 13,871.18. Locate the ending range associated with this value ($8,250.01 – $8,500.00). The third stratum range (boundary) is $2,200.01 – $8,500.00.

Step 13 Stratum 4 is whatever is left after calculating the stratum breaks for stratum1, 2, and 3.

APPENDIX

E

SAMPLE ALLOCATION

Sample Allocation

Once the sampling frame has been stratified, the sample size within each stratum needs to be determined. This can be done proportionately or "optimally".

Proportional sampling allocates the sample based solely on the strata sizes. Optimal allocation considers the size of the strata and the variability within each stratum.

In the following example of a stratified sampling frame, the initial population was refined to exclude items that were not of interest or detailed:

	COUNT	AMOUNT	AVERAGE	STD DEV	CV
TOTAL DOWNLOAD	27,478	64,852,762.11	2,360.17	33,964.35	1439.06%
SMALL INVOICES & NEGATIVES	8,957	(3,433,107.08)	(383.29)	20,572.87	-5367.48%
LARGE INVOICES	802	52,496,892.54	65,457.47	175,018.56	267.38%
EXCLUDED_EMP	1,036	223,236.18	215.48	252.32	117.10%
EXCLUDED_VEND	249	242,266.22	972.96	1,484.38	152.56%
DELETED VENDORS	1,857	1,368,113.13	736.73	1,336.31	181.38%
OTHER REFUND ITEMS	1,448	305,021.71	210.65	413.50	196.30%
OTHER INERCOMPANY	28	22,370.10	798.93	1,127.79	141.16%
SAMPLING FRAME	13,101	13,627,969.31	1,040.22	1,662.41	159.81%
	27,478	64,852,762.11			
STRATUM 1 (50 TO 874.99)	9,162	2,485,764.06	271.31	210.47	77.57%
STRATUM 2 (875 TO 3449.99)	2,877	4,961,067.32	1,724.39	670.92	38.91%
STRATUM 3 (3450 TO 10000)	1,062	6,181,137.93	5,820.28	1,809.21	31.08%
	13,101	13,627,969.31		Average ->	49.19%

The overall sample size is 900.

1. Proportional Allocation

The allocation of the sample to each stratum could be as follows:

DESCRIPTION	COUNT	%	SAMPLE DISTRIBUTION
STRATUM 1 (50 TO 874.99)	9,162	70%	629
STRATUM 2 (875 TO 3449.99)	2,877	22%	198
STRATUM 3 (3450 TO 10000)	1,062	8%	73
	13,101	100%	900

2. Optimal Allocation - Neyman

If optimal allocation is used, the sample size for each stratum could be as follows:

DESCRIPTION -A-	COUNT -B-	STANDARD DEVIATION (*see note*) -C-	COUNT X STD DEV (B * C) -D-	% of D -E-	SAMPLE DISTRIBUTION (E * 900) -F-
STRATUM 1 (50 TO 874.99)	9,162	210.47	1,928,326.14	33.36%	300
STRATUM 2 (875 TO 3449.99)	2,877	670.92	1,930,236.84	33.40%	301
STRATUM 3 (3450 TO 10000)	1,062	1,809.21	1,921,381.02	33.24%	299
	13,101		5,779,944.00	100.00%	900

note: This is the standard deviation of the book or invoice amounts. To really be "optimal", the distribution should be based on the standard deviation of the error amounts, which are unknown prior to sampling - so the best information available is used.

The Optimal allocation is the preferred method by the Commission. However, if strata breaks are determined using the Cumulative Square Root of the Frequency method, the sample is “approximately optimal” when the sample size within each stratum is equal. The allocation step can be ignored if strata breaks are determined using the Cumulative Square Root of the Frequency method and sample size is equal for each stratum. In this instance, sample size would be set at 300 items per stratum (900 divided by 3 strata). The minor variances from 300 (301 for stratum 2 and 299 for stratum 3) as noted in column F can be ignored.

APPENDIX

F

TERMS AND DEFINITIONS

Attribute – a qualitative characteristic of interest associated with sampling units.

Audited Amount – in financial auditing, is the amount that should be in the taxpayer's records.

Audit risk – is the risk that there is tax misstatement and that the auditor will not detect that error.

Block sample – a sample based on a portion of the population, commonly a large portion of the population, based on convenience and is a judgmental sample. Source records for block samples are generally located in one place or are physically continuous in nature.

Bootstrapping – statistical inference based on the results of repeatedly re-sampling the sample. This procedure can be used to establish a confidence interval from a probability sample without any assumptions regarding the sampling distribution.

Cluster sample – is a sample taken from a population divided into groups, or clusters. If a simple random sample is taken, where the cluster is the sampling unit, this type of sample is a probability sample. If the clusters are chosen using non-random methods, this would not be a probability sample. Regardless of the method of choosing the clusters, clusters chosen in the sample are then audited in their entirety.

Coefficient of variation (CV) – is the ratio of the standard deviation to the corresponding mean. The population CV measures the relative dispersion of the population distribution. The CV of an estimate is the standard error divided by the estimate.

Confidence interval – is the range of values between the lower confidence limit (LCL) and the upper confidence level (UCL) derived from the sample, which contains the true population value with a specified confidence level.

Confidence level – is the probability that the confidence interval will contain the true population amount that is of audit interest.

Correlation – is a measure of the degree in which two quantities are linearly related. This relationship could be one of dependence or association. Correlation in a sample or a population can be measured by the correlation coefficient, ρ. The extreme values for ρ of -1 or +1 signify an exact linear relationship. If ρ =0, then the two quantities have no linear relationship (a formula for deriving ρ is included in the endnotes).

Correspondence – the method of matching random numbers to sampling units from the sampling frame.

Decision rules – special valuation rules employed by the auditor in valuing the sample whereby taxable error has only one chance of being anything other than zero, avoiding selection bias and possible impairment of the sample results.

Detail – a review of all transactions within an audit by the auditor using thorough, complete, and consistent procedures throughout the examination. Statisticians often refer to a detail as a census.

Difference estimator – is an evaluation procedure in which the average taxable error (or other unknown) from the sample is used to estimate the average taxable error for the population. The total taxable error for the population is calculated by multiplying average taxable error from the sample by the population count.

Distribution – refers to the scattering or diffusion of values in a population.

Empirical Likelihood – is a form of statistical inference that can use to establish a confidence interval from a probability sample without any assumptions regarding the sampling distribution.

Error rate – the occurrence rate of sampling units in the population (or sample) that have a non-zero taxable error value.

Estimator – is the methodology used to estimate some unique value from a population.

Estimator Bias – is bias that occurs when the average value of the estimator (taken over all possible samples) is not equal to the population mean.

Frequency distribution – is a tabular representation of a population where the population is divided into classes or ranges and the number of population units falling into each class are counted and shown in the table.

Hypergeometric distribution – The exact sampling distribution for the sample occurrence rate when a simple random sample of a given sample size and population is selected. The distribution will be different for any change in the sample size or population size.

Judgment sample – any sample other than a probability sample which is picked from the population by the subjective decision of an individual and where the chance of selection is not known.

Lower confidence limit (LCL) – is the point estimate less the precision amount and is the smallest value in the confidence interval which contains the true population value with a specified confidence level.

Median – is the midpoint of either a sample or population.

Mean-per-unit estimator – an estimator that backs into the total taxable error value of the population by first estimating the total audited value for the population. The total taxable error value equals total invoice value less the total audited value. The total audited value is estimated by taking the average audited value from the sample multiplied by the population count. Each individual audit value for the sample is computed by subtracting the taxable error value for that item with the invoice value for that item.

Mode – is the value that occurs most often in the population.

Non-sampling error – an error that is encountered whether the population is sampled or not.

Normal distribution (Gaussian distribution) – is an important distribution in statistical theory used to estimate probabilities. It is symmetric and bell-shaped distribution and is the approximate sampling distribution for many statistical estimates.

Occurrence rate – the rate of occurrence in the population (or sample) of sample units exhibiting an attribute. The error rate represents the occurrence rate or proportion of sampling units in a population having a nonzero taxable error value.

Point estimate – is an estimate of a parameter of a population. Most often in tax auditing, the point estimate of interest is an estimate of total error.

Population – the aggregate of tax transactions about which information is desired.

Population mean – or the true mean value of the population.

Population standard deviation – is a measure of the dispersion about the population mean.

Precision – see *precision amount*.

Precision amount (precision) – is a measurement *sampling error* which is the measure of how close a sample estimate is from the corresponding population characteristic. It can be estimated from the sample results from a probability sample by multiplying the standard error of the estimate by a factor determined by the desired confidence.

Probability proportional to size sample (PPS) – is a probability sample where the chance of selection is proportional to the size (or dollar value) of the unit. Some references call this dollar units sampling (DUS) or monetary unit sampling (MUS). Mechanically, in PPS designs, each dollar in the population has an equal chance of selection. The randomization or matching to the random numbers is tied to each dollar in the population. If a particular dollar is selected into the sample, the entire document is pulled into the sample.

Probability sample – a sample where the chance of selection of every item in the population has a known, but not necessarily equal chance of selection (contrast this definition with that of a judgmental sample).

Range – the difference between the highest and lowest values in a group of items.

Ratio estimator – the population ratio, *R*, equals the total taxable error divided by the total invoice amount. The sample ratio, *r*, is used to estimate the population ratio. The sample ratio is derived by dividing the total taxable error in the sample by the total invoice amount in the sample. The sample ratio is used to estimate total taxable error by taking *r* and multiplying it by the known total population invoice value.

Regression estimator – uses the linear relationships of the taxable error values and the invoice values of the sample in addition to the known total population invoice value to estimate total taxable error in the population.

Relative precision (precision percentage) – is the precision amount expressed in relative terms to the point estimate.

Sample – is a part of the population.

Sample design (sampling plan) – a plan for sampling a population specified before sampling commences. Various sample designs include cluster sampling, multi-stage sampling, simple random sampling, and stratified random sampling. Key elements of the sample design include identification of the sampling frame and sampling unit, sample size, determination of the source of random numbers, definition of the characteristic being estimated, and identification of the estimator used to project the sample results.

Sample mean – equals the sum of the sample values divided by the sample size. The sample mean is an unbiased estimate of the population mean.

Sample standard deviation – is the measure of dispersion in the sample and an estimate of the population standard deviation.

Sampling error – is measured by the *precision amount* and is the difference between a value from a population, usually not known, and an estimate using a sample from that population.

Sampling frame (frame) – is the list or file sampled and is the means by which the target population is sampled.

Sampling distribution – is the distribution of all averages, totals, percentages, or other statistics for all possible samples at a given sample size for a certain population. If the sample is "large" then the sampling distribution will approximate the normal distribution.

Sampling risk – is the probability that the confidence interval will not contain the population parameter of interest.

Sampling unit – is each individual element of the sampling frame that can be selected into the sample.

Sampling with replacement – a sampling procedure where individual sampling units are returned to the population before selecting subsequent units.

Sampling without replacement – a sampling procedure where individual sampling units are not returned to the population before selecting subsequent units

Selection bias – occurs when a sampling unit has a probability of selection that is different from the planned probability.

Simple random sample – a specific form of probability sampling where if taken without replacement from a finite population, each possible sample for sample size *n* from a population of *N* is possible and has an equal chance of being selected.

Skew – a description or quality of a distribution that is asymmetrical, where the frequencies of the values are greater on one side or the other of the most frequent value.

Standard deviation – is a measure of variability within a population or a sample. The standard deviation is the square root of the variance.

Standard error – is the standard deviation of a sampling distribution and measures the variability of the estimates.

Statistical sampling - means any approach to sampling that has the following characteristics:

(1) Use of a probability sample; and

(2) Use of probability theory to evaluate sample results, including measurement of sampling risk.

A sampling approach that does not have characteristics (1) and (2) is considered non-statistical sampling.

Stratified random sample – is a form of probability sampling where a population is divided up into different groups, or strata. A simple random sample is taken from within each stratum.

Stratifying – dividing a population, or sampling frame, into groups, or strata. In most cases, stratifying is done to take more than one sample – although in some cases the auditor may stratify and take only one sample from one of the groups (which is not considered *stratified random sampling*).

Systematic Sample – is a sample of size *n* that is taken from a population of size *N* by selecting one (or more) *starting points* (where *k* equals the number of starting points), and then counting through the entire population and taken a specific unit that corresponds to the counting *interval* to fill the sample of at least size *n*. The counting interval is the largest integer less than or equal to *kN/n*. Starting points are generally some integer from 1 to the interval. Starting points may also be established from 1 to *N*, in which case the sampler must circle back to first population unit after counting through to the end of the population (unit *N*) to attain at least *n* units in the sample (*circular systematic sampling*). Where starting points are established using random selection techniques, that is selecting *k* random starting points 1 to the interval (or 1 to *N* for circular systematic sampling), this form of sampling is a probability sample.

Target population – is the population that is of audit interest.

Upper confidence limit (UCL) – is the point estimate plus the precision amount and is the largest value in the confidence interval which contains the true population value with a specified confidence level.

Variance – a measure of variability within a population or a sample. The variance is the standard deviation squared.

APPENDIX

G

NOTATIONS AND FORMULAS

The notation and formulas have been taken from a variety of sources, and if a reference can be provided, it is noted in the tables below. The Commission has endeavored to maintain as much consistency as possible with the primary source: Cochran's *Sampling Techniques*. The notation style is described in Cochran's book at pages 20-21 and again at 89-90. Uppercase letters refer to characteristics of a population and lowercase letters to those of a sample (see *Sampling Techniques*, page 20, last sentence).

The characteristic or parameter that is generally of interest to the sampler is defined as y or Y in *Sampling Techniques*. A correlated amount or other value is defined as x or X. Since sales tax auditors are almost always interested in total taxable error, the notation below uses y or Y to signify taxable error. An amount recorded in the taxpayer's records, generally for sales tax auditors some "invoice amount", will be designated with x or X (this is also used for "book value", "recorded amount", "reported amount", or "examined amount"). Most of the other sources cited below have elected not to follow this style, where the value of audit interest may be shown as X or some other letter. Therefore, when referencing these other sources, some substitution is required (note that the statistical functions in Excel include "Help" screens that contain notation that is generally consistent with *Sampling Techniques*).

Finally some borrowings are made from sources that provide formula for financial auditors. A concept heavily used in financial audits, the "audited value" has little meaning in sales tax auditing. Some of formulas cited will use the concept of audited value. These sources generally introduce yet another letter to signify the audited value. The Commission has elected not to follow this. Wherever these sources use audited value, the notation below will substitute either X-Y, x-y, or $\bar{x} - \bar{y}$ as it is known:

$$\textit{audited value} = \textit{recorded amount} - \textit{error}$$

General Notation:

Variable and/or Formula	Description	Source
N	Number of units in sampling frame	C p 20
n	Number of units in sample (sample size)	C p 20
$f = n/N$	Sampling fraction	C p 21
y_i	Taxable error value for a sampling unit from the sampling frame or sample, where i = 1, 2, …,n (if a from the sample), or i = 1, 2, …,N (if a from the population)	C p 20
x_i	Invoice value for a sampling unit from the sample or population	C p 150
$Y = \sum_{i=1}^{N} y_i = y_1 + y_2 + \cdots + y_N$	Total taxable error in the sampling frame	C p 20
$X = \sum_{i=1}^{N} x_i = x_1 + x_2 + \cdots + x_N$	Total invoice amount in the sampling frame	IRS Ls.Pg 3.5
$\overline{Y} = \frac{\sum_{i=1}^{N} y_i}{N}$	True mean taxable error in the sampling frame EXCEL FUNCTION: "=average(range of values)"	C p 20
$\overline{X} = \frac{\sum_{N}^{i=1} x_i}{N}$	True mean invoice amount in the sampling frame EXCEL FUNCTION: "=average(range of values)"	
$\hat{Y}$	Estimated total taxable error in the sampling frame (from an estimator)	C P 21
$\hat{X}$	Estimated total invoice amount in the sampling frame (from an estimator)	IRS Ls.Pg 3.5
$\sum_{i=1}^{n} y_i = y_1 + y_2 + \cdots + y_n$	Total taxable error in the sample	C p 20
$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$	Total invoice amount in the sample	
$\overline{y} = \frac{\sum_{i=1}^{n} y_i}{n}$	Mean taxable error in the sample EXCEL FUNCTION: "=average(range of values)"	C p 20
$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$	Mean invoice amount in the sample EXCEL FUNCTION: "=average(range of values)"	IRS Ls.Pg 3.5
$R = \frac{\overline{Y}}{\overline{X}} = \frac{Y}{X}$	True ratio of the taxable error to the total invoice amount in the population	
$\hat{R} = \frac{\overline{y}}{\overline{x}} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$	Estimated ratio of the taxable error to the total invoice amount from the sample	C p 21
L	Number of strata in the sampling frame sampled (excluding any strata detailed)	C p 89
$N_1 + N_2 + \cdots + N_L = \sum_{h=1}^{L} N_h = N$	Number of units in a stratified sampling frame where N_h is the number of sampling units in the hth stratum, where h = 1,2,…,L	C p 89

$n_1 + n_2 + \cdots + n_L = \sum_{h=1}^{L} n_h = n$	Number of units in a stratified random sample (sample size), where n_h is the sample size in the hth stratum.	C p 89
$f_h = n_h / N_h$	Sampling fraction in the hth stratum	C p 90
y_{hi}	The ith taxable error amount in the hth stratum of the sampling frame or sample	C p 90
x_{hi}	The ith invoice amount in the hth stratum of the sampling frame or sample	IRS Ls.Pg 10.14
$Y_h = \sum_{i=1}^{N_h} y_{hi}$	Total taxable error in the hth stratum of the sampling frame	
$X_h = \sum_{i=1}^{N_h} x_{hi}$	Total invoice amount in the hth stratum of the sampling frame	
$Y_{st} = \sum_{h=1}^{L} Y_h$	Total taxable error of a stratified sampling frame	
$X_{st} = \sum_{h=1}^{L} X_h$	Total invoice amount of a stratified sampling frame	
$\overline{Y}_h = \frac{\sum_{i=1}^{N_h} y_{hi}}{N_h}$	True mean taxable error in the hth stratum of the sampling frame EXCEL FUNCTION: "=average(range of values)"	C p 90
$\overline{X}_h = \frac{\sum_{i=1}^{N_h} x_{hi}}{N_h}$	True mean invoice amount in the hth stratum of the sampling frame EXCEL FUNCTION: "=average(range of values)"	
$\overline{y}_h = \frac{\sum_{i=1}^{n_h} y_{hi}}{n_h}$	Mean taxable error in sample of the hth stratum EXCEL FUNCTION: "=average(range of values)"	C p 90
$\overline{x}_h = \frac{\sum_{i=1}^{n_h} x_{hi}}{n_h}$	Mean invoice amount in sample of the hth stratum EXCEL FUNCTION: "=average(range of values)"	
$\hat{Y}_h$	Estimated total taxable error in the hth stratum of the sampling frame (from an estimator)	
$\hat{X}_h$	Estimated total invoice amount in the hth stratum of the sampling frame (from an estimator)	
$\hat{Y}_{st}$	Estimated total taxable error in a stratified sampling frame	
$\hat{X}_{st}$	Estimated total invoice amount in a stratified sampling frame	
$R_C = \frac{Y_{st}}{X_{st}} = \frac{\sum_{h=1}^{L} N_h \overline{Y}_h}{\sum_{h=1}^{L} N_h \overline{X}_h}$	True combined ratio of the taxable error to the total invoice amount in a stratified population	
$\hat{R}_C = \frac{\hat{Y}_{st}}{\hat{X}_{st}} = \frac{\sum_{h=1}^{L} N_h \overline{y}_h}{\sum_{h=1}^{L} N_h \overline{x}_h}$	Estimated combined ratio of the taxable error to the total invoice amount in a stratified population	R P 240
t	Student's t from a t-table EXCEL FUNCTION: "=tinv(2-tailed risk, degrees of freedom)"	C p 27
$n-1$	Degrees of freedom (un-stratified sampling)	C p 27

$n_e = \frac{\left(s_{\hat{Y},st}\right)^4}{\sum_{h=1}^{L}\left[\left(s_{\hat{Y}_h}\right)^4/\left(n_h-1\right)\right]}$	Effective degrees of freedom for stratified sampling, where $s_{\hat{Y},st}$ (the estimated standard error of the stratified sampling frame) and $s_{\hat{Y}_h}$ (the estimated standard error within the hth stratum) are from an estimator	C p 96 IRS Ls.Pg 10.16 10.18 10.21

Unstratified Difference Estimation:

Variable and/or Formula	Description	Source
$\hat{Y}_d = N\bar{y}$	Estimated total taxable error in the sampling frame (difference estimation)	C p 22
$s_y^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}$	Estimated variance of the taxable error EXCEL FUNCTION: "=var(range of values)"	C p 26
$s_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}}$	Estimated standard deviation of the taxable error EXCEL FUNCTION: "=stdev(range of values)"	C P 27
$s_{\hat{Y},d} = \frac{Ns_y\sqrt{1-f}}{\sqrt{n}}$	Estimated standard error of the total taxable error (difference estimation)	C (2.22)
$ts_{\hat{Y},d}$	Precision amount for the estimated total taxable error (difference estimation)	C P 27
$\hat{Y}_d \pm \left(ts_{\hat{Y},d}\right)$	Confidence interval for the estimated total taxable error (difference estimation)	C (2.24)

Unstratified Ratio Estimation:

Variable and/or Formula	Description	Source
$\hat{Y}_r = \hat{R}X$	Estimated total taxable error in the sampling frame (Ratio Estimation)	C (6.1)
$s_{y,r} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i-\hat{R}x_i\right)^2}{n-1}}$	Estimated standard deviation of the ratios	R P 238
$s_{\hat{Y},r} = \frac{Ns_{y,r}\sqrt{1-f}}{\sqrt{n}}$	Estimated standard error of the total taxable error (ratio estimation)	R P 238
$ts_{\hat{Y},r}$	Precision amount for the estimated total taxable error (ratio estimation)	C P 156
$\hat{Y}_r \pm ts_{\hat{Y},r}$	Confidence interval for the estimated total taxable error (ratio estimation)	C (6.14)

Unstratified Regression Estimation:

Variable and/or Formula	Description	Source
$b = \frac{\sum[(x_i - \bar{x})(y_i - \bar{y})]}{\sum(x_i - \bar{x})^2}$	Estimated regression coefficient EXCEL FUNCTION: "=slope(range of y_i values, range containing x_i values)"	C (7.19)
$\hat{Y}_g = N\bar{y} + b(X - N\bar{x})$	Estimated total taxable error in the sampling frame (regression estimation)	N (2.6)
$s_{y,g} = \sqrt{\frac{\sum_{i=1}^{n}[(y_i - \bar{y}) - b(x_i - \bar{x})]^2}{n-2}}$	Estimated standard deviation of the regression amounts EXCEL FUNCTION: "=steyx(range of y_i values, range containing x_i values)"	C P 195
$s_{\hat{Y},g} = \frac{Ns_{y,g}\sqrt{1-f}}{\sqrt{n}}$	Estimated standard error of the total taxable error (regression estimation)	R P 239
$ts_{\hat{Y},g}$	Precision amount for the estimated total taxable error (regression estimation)	H p 4-33
$\hat{Y}_g \pm ts_{\hat{Y},g}$	Confidence interval for the estimated total taxable error (regression estimation)	H p 4-33

Unstratified Mean-per-unit Estimation:

Variable and/or Formula	Description	Source
$\hat{Y}_m = X - N(\bar{x} - \bar{y})$	Estimated total taxable error in the sampling frame (mean-per-unit estimation)	N (2.2) (2.3)
$s^2_{(x-y)} = \frac{\sum_{i=1}^{n}[(x_i - y_i) - (\bar{x} - \bar{y})]^2}{n-1}$	Estimated variance of the audited amounts EXCEL FUNCTION: "=var(range of audited values)", where an audited value is equal to $x_i - y_i$	IRS Ls.Pg 5.8
$s_{(x-y)} = \sqrt{\frac{\sum_{i=1}^{n}[(x_i - y_i) - (\bar{x} - \bar{y})]^2}{n-1}}$	Estimated standard deviation of the audited amounts EXCEL FUNCTION: "=stdev(range of audited values)"	R P 237
$s_{\hat{Y},m} = \frac{Ns_{(x-y)}\sqrt{1-f}}{\sqrt{n}}$	Estimated standard error of the total taxable error (mean-per-unit estimation)	R P 237
$ts_{\hat{Y},m}$	Precision amount for the estimated total taxable error (mean-per-unit estimation)	IRS Ls.Pg 5.9
$\hat{Y}_m \pm ts_{\hat{Y},m}$	Confidence interval for the estimated total taxable error (mean-per-unit estimation)	IRS Ls.Pg 5.9

Stratified Difference Estimation:

Variable and/or Formula	Description	Source
$\hat{Y}_{st,d} = \sum_{h=1}^{L} N_h \bar{y}_h$	Estimated total taxable error in the stratified sampling frame (difference estimation)	R p 240
$s_{y_h} = \sqrt{\frac{\sum_{i=1}^{n_h} (y_{hi} - \bar{y}_h)^2}{n_h - 1}}$	Estimated standard deviation in the hth stratum EXCEL FUNCTION: "=stdev(range of values)"	IRS Ls.Pg 10.17
$s_{\hat{Y}_h,d} = \frac{N_h s_{y_h} \sqrt{1 - f_h}}{\sqrt{n_h}}$	Estimated standard error in the hth stratum (difference estimation)	IRS Ls.Pg 10.18
$s_{\hat{Y}_{st},d} = \sqrt{\sum_{h=1}^{L} s_{\hat{Y}_h,d}^2}$	Estimated standard error of the stratified sampling frame (difference estimation)	R p 240
$ts_{\hat{Y}_{st},d}$	Precision amount for the estimated total taxable error of the stratified sampling frame (difference estimation)	IRS Ls.Pg 10.18
$\hat{Y}_{st,d} \pm ts_{\hat{Y}_{st},d}$	Confidence interval for the estimated total taxable error in a stratified frame (difference estimation)	C (5.15)

Combined Ratio Estimation:

Variable and/or Formula	Description	Source
$\hat{Y}_{Rc} = \hat{R}_C X = \frac{\hat{Y}_{st}}{\hat{X}_{st}} X = \frac{\sum_{h=1}^{L} N_h \bar{y}_h}{\sum_{h=1}^{L} N_h \bar{x}_h} X$	Estimated total taxable error in the stratified sampling frame (combined ratio estimation)	C (6.48)
$s_{y_{Rc,h}} = \sqrt{\frac{\sum_{i=1}^{n_h} \left[(y_{hi} - \bar{y}_h) - \hat{R}_C (x_{hi} - \bar{x}_h) \right]^2}{n_h - 1}}$	Estimated standard deviation of ratio in the hth stratum	IRS Ls.Pg 10.20
$s_{Rc_h} = \frac{N_h s_{y_{Rc,h}} \sqrt{1 - f_h}}{\sqrt{n_h}}$	Estimated standard error in the hth stratum (combined ratio estimation)	IRS Ls.Pg 10.21
$s_{\hat{Y}_{Rc}} = \sqrt{\sum_{h=1}^{L} s_{Rc_h}^2}$	Estimated standard error of the stratified sampling frame (combined ratio estimation)	IRS Ls.Pg 10.21
$ts_{\hat{Y}_{Rc}}$	Precision amount for the estimated total taxable error of the stratified sampling frame (combined ratio estimation)	
$\hat{Y}_{Rc} \pm ts_{\hat{Y}_{Rc}}$	Confidence interval for the estimated total taxable error in a stratified frame (combined ratio estimation)	H p 4-21

Combined Regression Estimation:

Variable and/or Formula	Description	Source
$s_{x_h}^2 = \frac{\sum_{i=1}^{n_h}(x_{hi} - \bar{x}_h)^2}{n_h - 1}$	Estimated variance of the invoice amounts in h^{th} stratum	
$s_{y_h}^2 = \frac{\sum_{i=1}^{n_h}(y_{hi} - \bar{y}_h)^2}{n_h - 1}$	Estimated variance of the taxable errors in h^{th} stratum	
$\text{cov}(x,y)_h = \frac{\sum_{i=1}^{n_h}[(x_{hi} - \bar{x}_h)(y_{hi} - \bar{y}_h)]}{n_h - 1}$ *OR* $\text{cov}(x,y)_h = \frac{\left(\sum_{i=1}^{n_h} x_{hi} y_{hi}\right) - n_h \bar{x}_h \bar{y}_h}{n_h - 1}$	Estimated covariance between the taxable errors and the invoice amounts in the hth stratum EXCEL FUNCTION: "=$[n_h/(n_h-1)]$*covar(range of y_i values, range containing x_i values)"	R p 241
$b_c = \frac{\sum_{h=1}^{L}\left[N_h(N_h - n_h)\frac{\text{cov}(x,y)_h}{n_h}\right]}{\sum_{h=1}^{L}\left[N_h(N_h - n_h)\frac{s_{x_h}^2}{n_h}\right]}$	Estimated combined regression coefficient	H p 4-41 C P 202
$\hat{Y}_{Gc} = \hat{Y}_{st,d} + b_c\left(X - \hat{X}_{st}\right)$ *OR* $\hat{Y}_{Gc} = \left(\sum_{h=1}^{L} N_h \bar{y}_h\right) + b_c\left[X - \left(\sum_{h=1}^{L} N_h \bar{x}_h\right)\right]$	Estimated total taxable error in the stratified sampling frame (combined regression estimation)	H p 4-41
$s_{y_{Gc,h}} = \sqrt{s_{y_h}^2 - [2b_c \text{cov}(x,y)_h] + b_c^2 s_{x_h}^2}$	Estimated standard deviation of regression in the hth stratum	
$s_{Gc_h} = \frac{N_h s_{y_{Gc,h}} \sqrt{1 - f_h}}{\sqrt{n_h}}$	Estimated standard error in the hth stratum (combined regression estimation)	R p 241
$s_{\hat{Y}_{Gc}} = \sqrt{\sum_{h=1}^{L} s_{Gc_h}^2}$	Estimated standard error of the stratified sampling frame (combined regression estimation)	R p 241
$ts_{\hat{Y}_{Gc}}$	Precision amount for the estimated total taxable error of the stratified sampling frame (combined regression estimation)	
$\hat{Y}_{Gc} \pm ts_{\hat{Y}_{Gc}}$	Confidence interval for the estimated total taxable error in a stratified frame (combined regression estimation)	H p 4-42

Stratified Mean-per-unit Estimation:

Variable and/or Formula	Description	Source
$\hat{Y}_{st,m} = X - \sum_{h=1}^{L} N_h \left(\bar{x}_h - \bar{y}_h\right)$	Estimated total taxable error in the stratified sampling frame (mean-per-unit estimation)	IRS Ls.Pg 10.14 10.16
$s_{(x-y)_h} = \sqrt{\frac{\sum_{i=1}^{n_h}\left[\left(x_{hi} - y_{hi}\right) - \left(\bar{x}_h - \bar{y}_h\right)\right]^2}{n_h - 1}}$	Estimated standard deviation of the audited amounts in the hth stratum EXCEL FUNCTION: "=sdtev(range of audited values)", where an audited value is equal to $x_{hi} - y_{hi}$	IRS Ls.Pg 10.14
$s_{\hat{Y}_h,m} = \frac{N_h s_{(x-y)_h}\sqrt{1-f_h}}{\sqrt{n_h}}$	Estimated standard error in the hth stratum (mean-per-unit estimation)	IRS Ls.Pg 10.15
$s_{\hat{Y}_{st},m} = \sqrt{\sum_{h=1}^{L} s^2_{\hat{Y}_h,m}}$	Estimated standard error of the stratified sampling frame (mean-per-unit estimation)	IRS Ls.Pg 10.15
$ts_{\hat{Y}_{st},m}$	Precision amount for the estimated total taxable error of the stratified sampling frame (mean-per-unit estimation)	
$\hat{Y}_{st,m} \pm ts_{\hat{Y}_{st},m}$	Confidence interval for the estimated total taxable error in a stratified frame (mean-per-unit estimation)	

Separate Ratio Estimation:

Variable and/or Formula	Description	Source
$\hat{R}_h = \frac{\bar{y}_h}{\bar{x}_h}$	Estimated ratio of the taxable error from the sample in the hth stratum	H p 4-20
$\hat{Y}_{Rs} = \sum_{h=1}^{L} R_h X_h$	Estimated total taxable error in the stratified sampling frame (separate ratio estimation)	H p 4-20
$s_{y_{Rs,h}} = \sqrt{\frac{\sum_{i=1}^{n_h}\left(y_{hi} - \hat{R}_h x_{hi}\right)^2}{n_h - 1}}$	Estimated standard deviation of ratio in the hth stratum	H p 4-20
$s_{Rs_h} = \frac{N_h s_{y_{Rs,h}}\sqrt{1-f_h}}{\sqrt{n_h}}$	Estimated standard error in the hth stratum (separate ratio estimation)	H p 4-20
$s_{\hat{Y}_{Rs}} = \sqrt{\sum_{h=1}^{L} s^2_{Rs_h}}$	Estimated standard error of the stratified sampling frame (separate ratio estimation)	H p 4-20
$ts_{\hat{Y}_{Rs}}$	Precision amount for the estimated total taxable error of the stratified sampling frame (separate ratio estimation)	
$\hat{Y}_{Rs} \pm ts_{\hat{Y}_{Rs}}$	Confidence interval for the estimated total taxable error in a stratified frame (separate ratio estimation)	H p 4-20

Separate Regression Estimation:

Variable and/or Formula	Description	Source
$b_h = \frac{\sum_{i=1}^{n_h}\left[\left(x_{hi}-\bar{x}_h\right)\left(y_{hi}-\bar{y}_h\right)\right]}{\sum_{i=1}^{n_h}\left(x_{hi}-\bar{x}_h\right)^2}$	Estimated regression coefficient in the hth stratum EXCEL FUNCTION: "=slope(range of y_{hi} values, range containing x_{hi} values)"	C (7.56)
$\hat{Y}_{Gs} = \sum_{h=1}^{L}\left[N_h\bar{y}_h + b_h\left(X_h - N_h\bar{x}_h\right)\right]$	Estimated total taxable error in the stratified sampling frame (separate regression estimation)	R p 258
$s_{y_{Gs,h}} = \sqrt{\frac{\sum_{i=1}^{n_h}\left[\left(y_{hi}-\bar{y}_h\right)-b_h\left(x_{hi}-\bar{x}_h\right)\right]^2}{n_h-2}}$	Estimated standard deviation of regression in the hth stratum EXCEL FUNCTION: "=steyx(range of y_{hi} values, range containing x_{hi} values)"	H p 4-41
$s_{Gs_h} = \frac{N_h s_{y_{Gs,h}}\sqrt{1-f_h}}{\sqrt{n_h}}$	Estimated standard error in the hth stratum (separate regression estimation)	H p 4-41
$s_{\hat{Y}_{Gs}} = \sqrt{\sum_{h=1}^{L} s_{Gs_h}^2}$	Estimated standard error of the stratified sampling frame (separate regression estimation)	H p 4-41
$ts_{\hat{Y}_{Gs}}$	Precision amount for the estimated total taxable error of the stratified sampling frame (separate regression estimation)	
$\hat{Y}_{Gs} \pm ts_{\hat{Y}_{Gs}}$	Confidence interval for the estimated total taxable error in a stratified frame (separate regression estimation)	H p 4-41

Coefficient of Variation Calculations to Test for Estimator Bias (Unstratified):

Variable and/or Formula	Description	Source
$s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$	Estimated standard deviation of the invoice amounts EXCEL FUNCTION: "=stdev(range of values)"	
$s_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}}$	Estimated standard deviation of the taxable error EXCEL FUNCTION: "=stdev(range of values)"	
$s_{(x-y)} = \sqrt{\frac{\sum_{i=1}^{n}\left[\left(x_i-y_i\right)-\left(\bar{x}-\bar{y}\right)\right]^2}{n-1}}$	Estimated standard deviation of the audited amounts EXCEL FUNCTION: "=stdev(range of audited values)", where an audited value is equal to $x_i - y_i$	

$cv_x = \dfrac{Ns_x\sqrt{1-f}\,/\sqrt{n}}{N\bar{x}}$	Coefficient of variation of the estimated invoice amounts *(the denominator is the estimated total invoice value)*	C p 153
$cv_y = \dfrac{Ns_y\sqrt{1-f}\,/\sqrt{n}}{X - N\bar{y}}$	Coefficient of variation of the estimated taxable error amounts *(the denominator is the estimated total audited value using the difference estimator)*	C p 153
$cv_{(x-y)} = \dfrac{Ns_{(x-y)}\sqrt{1-f}\,/\sqrt{n}}{N(\bar{x}-\bar{y})}$	Coefficient of variation of the estimated audited amounts *(the denominator is the estimated total audited value using the mean-per-unit estimator)*	C p 153

To control bias and use ratio or regression estimators in an unstratified frame, a test must be applied to sample results where $cv_x \le 10\%$ **AND** either $cv_y \le 10\%$ or $cv_{(x-y)} \le 10\%$.[19] As unbiased estimators, the test does not apply to difference estimation or mean-per-unit estimation.

Coefficient of Variation Calculations to Test for Estimator Bias (Stratified):

Variable and/or Formula	Description	Source
$s_{x_h} = \sqrt{\dfrac{\sum_{i=1}^{n_h}(x_{hi}-\bar{x}_h)^2}{n_h-1}}$	Estimated standard deviation of the invoice amounts in the hth stratum EXCEL FUNCTION: "=stdev(range of values)"	
$s_{y_h} = \sqrt{\dfrac{\sum_{i=1}^{n_h}(y_{hi}-\bar{y}_h)^2}{n_h-1}}$	Estimated standard deviation of the taxable error in the hth stratum EXCEL FUNCTION: "=stdev(range of values)"	
$s_{(x-y)_h} = \sqrt{\dfrac{\sum_{i=1}^{n_h}\left[(x_{hi}-y_{hi})-(\bar{x}_h-\bar{y}_h)\right]^2}{n_h-1}}$	Estimated standard deviation of the audited amounts in the hth stratum EXCEL FUNCTION: "=stdev(range of audited values)", where an audited value is equal to $x_{hi} - y_{hi}$	
$cv_{x_{st}} = \dfrac{\sqrt{\sum_{h=1}^{L}\left(N_h s_{x_h}\sqrt{1-f_h}\,/\sqrt{n_h}\right)^2}}{\sum_{h=1}^{L}(N_h\bar{x}_h)}$	Coefficient of variation of the estimated invoice amounts in a stratified frame *(the denominator is the estimated total invoice value for the entire frame)*	C p 153
$cv_{y_{st}} = \dfrac{\sqrt{\sum_{h=1}^{L}\left(N_h s_{y_h}\sqrt{1-f_h}\,/\sqrt{n_h}\right)^2}}{\sum_{h=1}^{L}(X_h - N_h\bar{y}_h)}$	Coefficient of variation of the estimated taxable error amounts in a stratified frame *(the denominator is the estimated total audited value for the entire frame using the difference estimator)*	C p 153

$cv_{(x-y)_{st}} = \dfrac{\sqrt{\sum_{h=1}^{L}\left(N_h s_{(x-y)_h}\sqrt{1-f_h}\Big/\sqrt{n_h}\right)^2}}{\sum_{h=1}^{L}\left[N_h\left(\bar{x}_h-\bar{y}_h\right)\right]}$	Coefficient of variation of the estimated audited amounts in a stratified frame *(the denominator is the estimated total audited value for the entire frame using the mean-per-unit estimator)*	C p 153
$cv_{x_h} = \dfrac{N_h s_{x_h}\sqrt{1-f_h}\Big/\sqrt{n_h}}{N_h\bar{x}_h}$	Coefficient of variation of the estimated invoice amounts in the hth stratum *(the denominator is the estimated total invoice value in the hth stratum)*	C p 153
$cv_{y_h} = \dfrac{N_h s_{y_h}\sqrt{1-f_h}\Big/\sqrt{n_h}}{X_h - N_h\bar{y}_h}$	Coefficient of variation of the estimated taxable error amounts in the hth stratum *(the denominator is the estimated total audited value in the hth stratum using the difference estimator)*	C p 153
$cv_{(x-y)_h} = \dfrac{N_h s_{(x-y)_h}\sqrt{1-f_h}\Big/\sqrt{n_h}}{N_h\left(\bar{x}_h-\bar{y}_h\right)}$	Coefficient of variation of the estimated audited amounts in the hth stratum *(the denominator is the estimated total audited value in the hth stratum using the mean-per-unit estimator)*	C p 153

To control bias and use combined ratio or combined regression estimators in a stratified frame, a test must be applied to sample results where $cv_{x_{st}} \le 10\%$ **AND** either $cv_{y_{st}} \le 10\%$ or $cv_{(x-y)_{st}} \le 10\%$. Where separate ratio and separate regression are considered in a stratified frame, the test is by stratum h and must be applied individually to each of the sampled strata (h=1, 2 …L) such that $cv_{x_h} \le 10\%$ **AND** either $cv_{y_h} \le 10\%$ or $cv_{(x-y)_h} \le 10\%$ pass for all sampled strata. As unbiased estimators, the test does not apply to stratified difference estimation or stratified mean-per-unit estimation.[20]

Classic Formulary Approach to Sample Size:

Variable and/or Formula	Description	Source
z EXCEL FORMULA: "=normsinv(1-sided confidence)"	The normal deviate derived from the standard normal table, such as for the following 2-sided confidence levels: 50% .67; 60% .84; 80% 1.28; 90% 1.64; 95% 1.96	C p 27
$n_{\Pr}$	Sample Size of the probe sample	
$drp\%$	Desired relative precision	

$n_{\Pr_h}$	Sample Size of the probe sample in the hth stratum	
$\hat{Y}_{\Pr} = N \dfrac{\sum_{i=1}^{n_{\Pr}} y_i}{n_{\Pr}}$	Estimated total taxable error using a probe sample in an unstratified frame	
$\hat{Y}_{\Pr} = \sum_{h=1}^{L} \left(N_h \dfrac{\sum_{i=1}^{n_{(\Pr)_h}} y_{hi}}{n_{\Pr_h}} \right)$	Estimated total taxable error using a probe sample in a stratified frame	
$\hat{s}^2 = \dfrac{\sum_{i=1}^{n_{\Pr}} (y_i - \bar{y})^2}{n_{\Pr} - 1}$	Variance of the sample estimated from a probe sample EXCEL FUNCTION: "var(range of values)"	
$\hat{s}_h^2 = \dfrac{\sum_{i=1}^{n_{\Pr_h}} (y_{hi} - \bar{y}_h)^2}{n_{\Pr_h} - 1}$	Variance of the sample estimated from a probe sample in the hth stratum EXCEL FUNCTION: "var(range of values)"	
$\hat{s}_h = \sqrt{\hat{s}_h^2}$	Standard deviation of the sample estimated from a probe sample in the hth stratum EXCEL FUNCTION: "stdev(range of values)"	
$d = drp\% * \hat{Y}_{\Pr}$	Margin of error (error tolerance)	C p 105
$n = \dfrac{N^2 z^2 \hat{s}^2}{d^2 + \left(N z^2 \hat{s}^2\right)}$	Computed sample size for an unstratified sampling frame (Formula I)	R p 238
$n = \dfrac{\left(\hat{s}N\right)^2}{\left(N\hat{s}^2\right) + \left(d/z\right)^2}$	Computed sample size for an unstratified sampling frame (Formula II)	
$n = \dfrac{\hat{s}^2 N}{\hat{s}^2 + \left\{N\left[drp\% * \left(\hat{Y}_{\Pr}/N\right)/z\right]^2\right\}}$	Computed sample size for an unstratified sampling frame (Formula III)	
$n = \dfrac{z^2\left(\sum_{h=1}^{L} N_h \hat{s}_h\right)^2}{d^2 + \left[z\left(\sum_{h=1}^{L} N_h \hat{s}_h^2\right)\right]}$	Computed sample size for a stratified sampling frame (n must be then allocated to the strata) (Formula IV)	R p 240

Note: sample size formulas I, II, and III for an unstratified frame should provide the same result.

Sample Size Calculations – Error Rate Model I:

Variable and/or Formula	Description					Source
Z EXCEL FORMULA: "=normsinv(1-sided confidence)"	The normal deviate derived from the standard normal table, such as for the following 2-sided confidence levels:					C p 27
	50%	60%	80%	90%	95%	
	.67	.84	1.28	1.64	1.96	

$p = \frac{a}{n}$	Error rate of the sample where the proportion a/n from a sample size n where a is the total number of sample units where $y_i \neq 0$	
$\hat{p}$	An estimate of the sample error rate, p	
$drp\%$	Desired relative precision	
$\hat{Y}_{eI} = X\hat{p}$	A rough estimate of the total taxable error in a sampling frame *(this should alternatively be determined through difference estimation – if feasible)*	
$d = \hat{Y}_{eI} drp\%$	Margin of error (error tolerance)	C p 105
$\overline{X} = \frac{\sum_{i=1}^{N} x_i}{N}$	True mean invoice amount in the sampling frame EXCEL FUNCTION: "=average(range of values)"	
$\overline{X}_h = \frac{\sum_{i=1}^{N_h} x_{hi}}{N_h}$	True mean invoice amount in the hth stratum of the sampling frame EXCEL FUNCTION: "=average(range of values)"	
$S_x^2 = \frac{\sum_{i=1}^{N} (x_i - \overline{X})^2}{N}$	Variance of the invoice amounts EXCEL FUNCTION: "=varp(range of values)"	
$S_{x_h}^2 = \frac{\sum_{i=1}^{N_h} (x_i - \overline{X})^2}{N_h}$	Variance of the invoice amounts in the hth stratum EXCEL FUNCTION: "=varp(range of values)"	
$\hat{s}_{eI}^2 = \hat{p}\left[S_x^2 + (1-\hat{p})\overline{X}^2\right]$	Estimated sample variance of the taxable error	R p 75
$\hat{s}_{eI_h}^2 = \hat{p}\left[S_{x_h}^2 + (1-\hat{p})\overline{X}_h^2\right]$	Estimated sample variance of the taxable error in the hth stratum *(note that $\hat{p}$ can be varied by stratum, if that information is available)*	R p 75
$\hat{s}_{eI_h} = \sqrt{\hat{s}_{eI_h}^2}$	Estimated sample standard deviation of the taxable error in the hth stratum	
$n = \frac{N^2 z^2 \hat{s}_{eI}^2}{d^2 + \left(Nz^2 \hat{s}_{eI}^2\right)}$	Computed sample size for an unstratified sampling frame (Formula I) assuming that: 1. All non-zero taxable error values are either all positive, or all negative. 2. All taxable error values are 100% of the absolute value of the invoice amount. 3. The variation in the non-zero errors only is approximately the same as the variation in the corresponding invoice values *Note that similar substitutions can be made for Formulas II and III*	

$n = \dfrac{z^2\left(\sum_{h=1}^{L} N_h \hat{s}_{eI_h}\right)^2}{d^2 + \left[z\left(\sum_{h=1}^{L} N_h \hat{s}^2_{eI_h}\right)\right]}$	Computed sample size for a stratified sampling frame (*n* must be then allocated to the strata) (Formula IV) with the same assumptions as stated for an unstratified frame	

Sample Size Calculations – Error Rate Model II:

Variable and/or Formula	Description	Source
π	Proportion of the nonzero items, *a*, in the sample greater than zero	
$\hat{\pi}$	An estimate of the proportion of the nonzero items, *a*, in the sample greater than zero *(note that error rate model II cannot compute a result if* $\hat{\pi} = .5$*)*	
$\hat{Y}_{eII} = X\hat{p}\hat{\pi} - X\hat{p}(1-\hat{\pi})$	A rough estimate of the total taxable error in a sampling frame *(this should alternatively be determined through difference estimation – if feasible)*	
$d = \hat{Y}_{eII} drp\%$	Margin of error (error tolerance)	
$\hat{s}^2_{eII} = \left(\hat{p}S^2_x\right) + \left[\hat{p}(1-\hat{p})\overline{X}^2\right] + \left[4\hat{p}\hat{\pi}(1-\hat{\pi})\overline{X}^2\right]$	Estimated sample variance of the taxable error	KR
$\hat{s}^2_{eII_h} = \left(\hat{p}S^2_{x_h}\right) + \left[\hat{p}(1-\hat{p})\overline{X}^2_h\right] + \left[4\hat{p}\hat{\pi}(1-\hat{\pi})\overline{X}^2_h\right]$	Estimated sample variance of the taxable error in the hth stratum *(note that* $\hat{p}$ *and* $\hat{\pi}$ *can be varied by stratum, if that information is available)*	KR
$\hat{s}_{eII_h} = \sqrt{\hat{s}^2_{eII_h}}$	Estimated sample standard deviation of the taxable error in the hth stratum	KR
$n = \dfrac{N^2 z^2 \hat{s}^2_{eII}}{d^2 + \left(Nz^2\hat{s}^2_{eII}\right)}$	Computed sample size for an unstratified sampling frame (Formula I) assuming that: 1. All taxable error values are 100% of the absolute value of the invoice amount 2. The variation in the non-zero errors only is approximately the same as the variation in the corresponding invoice	

	values *Note that similar substitutions can be made for Formulas II and III*	
$n = \frac{z^2\left(\sum_{h=1}^{L} N_h \hat{s}_{eII_h}\right)^2}{d^2 + \left[z\left(\sum_{h=1}^{L} N_h \hat{s}^2_{eII_h}\right)\right]}$	Computed sample size for a stratified sampling frame (*n* must be then allocated to the strata) (Formula IV) with the same assumptions as stated for an unstratified frame	

Probabilities for Seeing a Minimum Number of Non-zero Errors in the Sample given a Certain Error Rate in the Frame (Hypergeometric Distribution):

Variable and/or Formula	**Description**	**Source**
A	Number of sampling units in the sampling frame where $y_i \neq 0$	Cp 50
A'	Number of sampling units in the sampling frame where $y_i = 0$	Cp 50
$P = A/N$	Total proportion of sampling units in the sample frame size *N* where $y_i \neq 0$	Cp 50
a	Number of sampling units in the sample where $y_i \neq 0$	Cp 50
a'	Number of sampling units in the sample where $y_i = 0$	Cp 50
$p = a/n$	Total proportion of sampling units in the sample size *n*	Cp 50

	where $y_i \neq 0$	
$\left[\frac{n!}{a!(n-a)!}\right]\left[\frac{A(A-1)\ldots(A-a+1)(A')(A'-1)\ldots(A'-a'+1)}{N(N-1)\ldots(N-n+1)}\right]$ EXCEL FUNCTION: "=hypgeodist(*a*, *n*, *A*, *N*)"	Hypergeometric probability of finding a units in the sample	C (3.16)
$\Pr(a=0) =$ *(fraction #1)(fraction #2)(fraction #3)… (fraction #n)* Where *n* is the sample size, and there are *n* fractions where both the numerator and denominator in each fraction contain bracketed right-hand terms beginning with "(0)" in the first fraction (#1) and "(*n*-1)" in the last fraction (#*n*): $\text{fraction \#1} = \frac{N-A-(0)}{N-(0)}$ $\text{fraction \#2} = \frac{N-A-(1)}{N-(1)}$ $\text{fraction \#3} = \frac{N-A-(2)}{N-(2)}$ $\text{fraction \#}n = \frac{N-A-(n-1)}{N-(n-1)}$ EXCEL FUNCTION: "=hypgeodist(0, *n*, *A*, *N*)"	Hypergeometric probability of finding an error free sample, where *a*=0 for a given *P*	D p 65
$\Pr(a=1) = \frac{\Pr(a=0)}{[N-n-A+(1)][A+1-(1)][n+1-(1)](1)}$ EXCEL FUNCTION: "=hypgeodist(1, *n*, *A*, *N*)"	Hypergeometric probability computation for finding ***exactly*** one error (a=1) in the sample, if it is also mathematically possible to have *a*=0 in the sample for a given *P*	
$\Pr(a=2) = \frac{\Pr(a=1)}{[N-n-A+(2)][A+1-(2)][n+1-(2)](2)}$ EXCEL FUNCTION: "=hypgeodist(2, *n*, *A*, *N*)"	Hypergeometric probability computation for finding ***exactly*** two errors (a=2) in the sample, if it is also mathematically possible to have *a*=1 in the sample for a given *P*	
$\Pr(a=3) = \frac{\Pr(a=2)}{[N-n-A+(3)][A+1-(3)][n+1-(3)](3)}$	Hypergeometric probability computation for finding ***exactly*** three errors	

EXCEL FUNCTION: "=hypgeodist(3, *n*, *A*, *N*)"	(a=3) in the sample, if it is also mathematically possible to have ***a***=2 in the sample for a given ***P***	
$\Pr(a=4)=\dfrac{\Pr(a=3)}{[N-n-A+(4)][A+1-(4)][n+1-(4)](4)}$ EXCEL FUNCTION: "=hypgeodist(4, *n*, *A*, *N*)"	Hypergeometric probability computation for finding ***exactly*** four errors (a=4) in the sample, if it is also mathematically possible to have ***a***=3 in the sample for a given ***P***	
$\Pr(a \geq 1)=1-\Pr(a=0)$	Hypergeometric probability of finding ***at least*** one error in the sample (a≥1)	
$\Pr(a \geq 3)=1-\Pr(a=0)-\Pr(a=1)-\Pr(a=2)$	Hypergeometric probability of finding ***at least*** three errors in the sample (a≥3)	
$\Pr(a \geq 5)=1-\Pr(a=0)-\Pr(a=1)-\Pr(a=2)-\Pr(a=3)-\Pr(a=4)$	Hypergeometric probability of finding ***at least*** five errors in the sample (a≥5)	

Estimating Skew of the Taxable Error Amounts from the Invoice Amounts (Unstratified):

Variable and/or Formula	Description	Source
$\overline{X}=\dfrac{\sum_{N}^{i=1} x_i}{N}$	True mean invoice amount in the sampling frame EXCEL FUNCTION: "=average(range of values)"	
$S_x=\sqrt{\dfrac{\sum_{i=1}^{N}(x_i-\overline{X})^2}{N}}$	Standard deviation of the invoice amounts in the sampling frame EXCEL FUNCTION: "=stdevp(range of values)"	

$G_{1x} = \dfrac{\sum_{i=1}^{N}\left[\left(x_i - \overline{X}\right)^3\right]}{N\left(S_x\right)^3}$	(Fisher's measure of) skew of the invoice amounts EXCEL FUNCTION: $\dfrac{(N-2)g_1}{\sqrt{[N(N-1)]}}$ where g_1 is a measure of skew (of a sample) according to: "=skew(range of values)"	C (2.70) R P 236
$\hat{P}$	Estimated error rate of a sampling frame which estimates the proportion A/N from a sampling frame size N where A is the total number of sample units where $y_i \neq 0$	
$\hat{G}_{1y} = \dfrac{\left\{\left(S_x\right)^2\left[\left(S_x\right)G_{1x} + 3\overline{X}\left(1-\hat{P}\right)\right]\right\} + \left[\overline{X}^3\left(1-\hat{P}\right)\left(1-2\hat{P}\right)\right]}{\left[\left(S_x\right)^2 + \left(1-\hat{P}\right)\overline{X}^2\right]\sqrt{\hat{P}\left[\left(S_x\right)^2 + \left(1-\hat{P}\right)\overline{X}^2\right]}}$	Estimated skew of the taxable error amounts in a sampling frame assuming: 1. All non-zero taxable error values are either all positive, or all negative. 2. All taxable error values are 100% of the absolute value of the invoice amount. 3. The skew in the non-zero errors only is approximately the same as the skew in the corresponding invoice values	Kv

Sources:

C: *Sampling Techniques*, William Cochran, John Wiley and Sons, 1977

Recorded value (invoice amount) (total & element): X & x_i

Difference amount (taxable error value) (total & element): Y & y_i

Audited value (total & element): Y & y_i

D: *Dollar-unit Sampling, A Practical Guide for Auditors*, Donald A Leslie, Albert D Teitlebaum & Rodney J. Anderson, Chicago, CCH 1979

H: *RAT-STATS Companion Manual*, Department of Health and Human Services, Office of Inspector General, Office of Audit Services, Summer of 1996

Recorded value (invoice amount) (total & element): T_x & x

Difference amount (taxable error value) (total & element): T_d & d_i

Audited value (total & element): T_y & y_i

IRS: *Advanced Statistical Sampling, Student Coursebook*, Internal Revenue Service, May 1992 (Training 3174-002, TPDS 87030A)

Recorded value (invoice amount) (total & element): Y & y_i

Difference amount (taxable error value) (total & element): D & d_i

Audited value (total & element): X & x_i

KR: *Model of Sales & Use Tax File*, Donald Roberts and Richard Kulp, 2006, unpublished paper

Kv: *Estimating the Skew of the Taxable Error Population Using the Invoice Amounts*, Allan Kvanli, 2005, unpublished paper

N: *Accounting Estimates by Computer Sampling*, Maurice S Newman, John Wiley & Sons, New York, 1982

Recorded value (invoice amount) (total & element): X & x_i

Difference amount (taxable error value) (total): X-Y

Audited value (total & element): Y & y_i

R: *Statistical Auditing*, Donald A Roberts, AICPA, 1978

Recorded value (invoice amount) (total & element): Y & y_i

Difference amount (taxable error value) (total & element): D & d_i

Audited value (total & element): X & x_i

END NOTES

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[1] Note Arkin's discussion of "extraneous units" in *Sampling Methods for the Auditor*, page 145.

[2] If we have a sampling frame that is broken up into, say four strata, with a single overall projection across the four strata, this would count as one projection – not four.

[3] An error can be a debit (deficiency) or a credit (overpayment).

[4] In non-statistical sampling, sample size will never go below 250, but will probably be much greater than 300 according to the formula shown in section 0406.20 (transaction sampling).

The minimum of 300 for statistical sampling is needed to insure a reasonable chance of finding errors in the each of the strata, if significant error exists in the population.

[5] For audits where the electronic file is unavailable, the standard deviation of the invoice amounts can be estimated by the following formula:

$$\frac{\text{(High Invoice Amount in the Frame – Low Invoice Amount in the Frame)}}{4}$$

The average can be estimated by multiplying the estimated error rate by the total invoice amount divided by the total number of invoices.

6 Because mean-per-unit estimation performs poorly in low error rate populations, this estimator will be effectively ignored for populations where the error-rate is less than 20%. This means that for samples where the error rate is small, this estimator will never be used.

[7] A correlation coefficient, ρ, can easily be computed in Excel with the function "=correl([range of x_i values],[range of y_i values]", where the x_i values represent the invoice amounts in the population (or sample), and the y_i values represent the taxable error values in the population (or sample). The formula for correlation is:

$$\rho = \frac{\sum_{i=1}^{n}\left((x_i - \bar{x}) * (y_i - \bar{y})\right)}{\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2 * \sum_{i=1}^{n}(x_i - \bar{x})^2}}.$$

8 In effort to control bias in ratio and regression estimation, and the fact that the estimated standard error calculation for ratio estimation is approximate, the conditions noted here should be followed. Refer to Cochran's *Sampling Techniques* at pages 153 and 166.

[9] The basis for the concern is found in Cochran's book, *Sampling Techniques*, at pages 153 and166. He recommends that the coefficient of variation of the

sample mean for the invoice amounts should be less than 10%. Application of Cochran's concern over bias has been extended to the coefficients of variation of the sample mean for the audited amounts using both mean-per-unit and difference estimation.

[10] The combined approach is preferred because there is more concern over bias in separate estimation than in combined estimation as per Cochran's book, *Sampling Techniques*, page 166: "The combined approach is much less subject to the risk of bias than the separate estimate". According to Dr. Roberts' book, *Statistical Auditing*, the combined methodology is the commonly used approach in auditing applications, page 108.

11 The Commission does not use other approaches, such as the bootstrap and empirical likelihood to calculate a confidence interval.

[12] In random sampling, every sample of the size of *n* should be equally probable. Indiscriminately removing items may violate this basic rule unless the item is removed from both the sample and the population.

[13] Refer to the *Handbook of Sampling for Auditing and Accounting*, page 21.

[14] Arkin has a discussion of the proper treatment of missing items in *Sampling Methods for the Auditor*, pages 145-147.

[15] The following are some of the references:

1. *Statistical Auditing*, Donald Roberts, AICPA, 1978, pages 97 to 98.
2. *Sampling Techniques,* William Cochran, John Wiley and Sons, 1977, pages 127-131.
3. *Applications of Statistical Sampling to Auditing, Alvin Arens & James K Loebbecke, Prentice* Hall, Englewood CA 1981, pages 256 – 258.
4. IRS training publication: *Advanced Statistical Sampling*, Training 3174-002 Rev (05-92) TPDS 87030A in Lesson 10, pages 6 – 10.
5. *Survey Sampling*, Leslie Kish, John Wiley & Sons, 1995, pages 104 – 106.
6. *Sampling of Populations: Methods and Applications*, 3rd Edition, Paul Levy and Stanley Lemeshow, John Wiley & Sons, New York, 1999, pages 179 - 183.
7. *Elementary Survey Sampling*, 5th edition, Richard Scheaffer, William Mendenhall, R. Lyman Ott, Duxbury Press, New York, 1996, page 165.

[16] According to *Elementary Survey Sampling*, 5th edition, Richard Scheaffer et al. footnote13, page 189, CSRF was developed by T. Dalenius and J.L. Hodges Jr.

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[17] Generally, there is a relationship between error and invoice amounts. We know that the variation in the errors has something to do with the range of values in the non-zero error amounts. However, the biggest contributor to the variation of errors is usually the number of zero errors as percentage of the total (the more zero errors, the more variable the population). The number of zero errors in the population will not have anything to do with variation in values of invoice amounts. CSRF will not consider the effect of the zeros.

[18] In *Statistical Auditing*, Roberts describes the methodology of unequal lengths in a footnote on page 98. In *Sampling Techniques*, Cochran describes this modification on page 130.

[19] In addition to controlling bias, these tests should be applied so that the assumption of a "large-sample" is valid and the estimates of variance for ratio and regression are usable. The estimated standard error formulas included in the notation for ratio are only approximations, and are usable only if a large-sample is obtained according to Cochran, *Sampling Techniques* at page 153.

[20] In addition to controlling bias, these tests should be applied so that the assumption of a "large-sample" is valid and the estimates of variance for combined ratio and combined regression are usable. The estimated standard error formulas included in the notation for ratio are only approximations, and are usable only if a large-sample is obtained according to Cochran, *Sampling Techniques* at page 153. If the separate methods are not usable as a result of failing a cv test within one or more of the strata, the combined ratio and combined regression may be still usable provided the stratified cv tests pass.