Applying a Regression Analysis to the CUP Method

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The comparable uncontrolled price (CUP) method is considered the most reliable test to determine whether a transfer occurs at arm's length, since it is based on transactional information. In applying the CUP method, transactions must be comparable in property or service, contractual terms, and geographic market.  

Comparability may pose a greater problem when the taxpayer has multiple transactions during a given tax year. As in the quotation above from Dante's Inferno, if you have a CUP, then you are in compliance with the law and face no penalty.

This article offers a method to test the existence of a CUP under facts and circumstances involving multiple transactions. The method is based on regression analysis, which is capable of producing extremely reliable results. The article uses data from a case that was settled with the Internal Revenue Service at the examination level. Since the CUP analysis presented could not be refuted under audit examination, the IRS withdrew the proposed tax deficiency and its related penalties. Certain details are omitted to prevent the disclosure of confidential information.

Comparable Uncontrolled Prices

The OECD guidelines and IRS transfer pricing regulations generally regard the CUP method as the best method to evaluate the arm's-length character of a controlled transaction. 2

Regression Model

This article proposes a simple regression model to establish a CUP:

1 See Organization for Economic Cooperation and Development, Transfer Pricing Guidelines For Multinational Enterprises and Tax Administrations (Paris, 1995), §§2.7 and 2.8. See also Regs. §1.482-3(b)(2)(ii) (Comparability under the CUP method).

2 For example, Regs. §1.482-3(b)(ii) provides as follows: "The results derived from applying the comparable uncontrolled price method generally will be the most direct and reliable measure of an arm's-length price for the controlled transaction if an uncontrolled transaction has no differences with the controlled transaction that would affect the price, or if there are only minor differences that have a definite and reasonably ascertainable effect on price and for which appropriate adjustments are made."

Likewise, the OECD guidelines at §2.7 state: "Where it is possible to locate comparable uncontrolled transactions, the CUP method is the most direct and reliable way to apply the arm's-length principle. Consequently, in such cases the CUP Method is preferable over all other methods."

The OECD guidelines further say at §2.9: "[T]he difficulties that arise in attempting to make reasonably accurate adjustments [to eliminate the effect on price of the differences between the transactions being compared] should not routinely preclude the possible application of the CUP method."
\[(1) \quad y_i = \beta x_i + \epsilon_i,\]

under the hypothesis that the slope coefficient $\beta = 1$.

In the regression model above, the abscissa (or the x axis) represents the price charged by the taxpayer to uncontrolled customers (where each $x_i$ represents a different product price); and the ordinate (or the y axis) represents the price charged by the taxpayer to the controlled party (where each $y_i$ represents a different product price). 3 The variable $\epsilon$ represents a random error, assumed to have a constant variance.

3 Regs. §1.482-3(b)(ii)(B)(5)(i) (Indirect evidence of comparable uncontrolled transactions) provide that under specific circumstances a CUP can be derived from data from public exchanges or quotation media. In such circumstances, equation (1) can be transformed from a cross-section into a time-series:

\[(1a) \quad y_t = \beta w_t + v_t,\]

where $w_t$ denotes a price from the quoted media during period $t$, and $v_t$ denotes the associated random error. Regression (1a) can be fitted with an intercept, under the assumption that the intercept is not different from zero. In practice, the analyst must test and correct for correlated errors.

When the slope coefficient is at $\beta = 1$ (or is not statistically different from one), the two price series are identical, except for random errors. This result can be corroborated with an acceptable statistical confidence, and the taxpayer can thus establish a CUP regarding $y_i$, for all $i = 1$ to $N$ transactions.

**Exemplum**

The purpose of using regression analysis is to fit a line through a scatter of coordinated pairs of controlled and comparable uncontrolled prices in order to determine if such prices are statistically the same.

**Background**

For the purpose of the exemplum, assume that during a given tax year under audit, a foreign parent exported to its U.S. distributors tangible products. The foreign parent charged $y_i$ to the controlled U.S. distributor and charged $x_i$ to uncontrolled U.S. distributors. Diligence showed no quantity discounts or any other dissimilar contractual arrangement between the controlled and uncontrolled transactions. The only identifiable difference between the controlled and the uncontrolled transactions was certain difference in date of shipment during the applicable tax year.

Therefore to control for potential shipment date differences, which could be material, every price paid by the controlled and the uncontrolled parties was deflated or normalized by the daily quoted price of a basic commodity (a component used in producing the products), using the date of shipment of the respective products. 4 In addition, the original export prices were converted into U.S. dollars by using the daily exchange rate corresponding to the date of shipment.

4 When comparing prices of the same product shipped to controlled and uncontrolled distributors on different dates during the same tax year, an adjustment was made according to the formula:

\[x_i = X_{i, t} / Z_t, \quad \text{and} \quad y_i = Y_{i, t} / Z_t,\]

where $x$ or $y$ represents the adjusted price of product $i$ used in the regression analysis, $X_{i, t}$ or $Y_{i, t}$ represents the invoiced price of product $i$ on day $t$ (date of product shipment), and the deflator $Z_t$ represents the daily spot price of the commodity quoted in the metals exchange market.

**Regression Results**

A large number of invoices exceeding 20,000 transactions per year had to be analyzed. For the purpose of illustrating the method proposed herein, one of several years was selected to identify the subset pairs of controlled and uncontrolled prices exchanged in that year. 5 See the Appendix for guidance regarding preparation of the CUP dataset. Next, a least squares regression line was fitted through the coordinated ($x_i$, $y_i$) pairs of normalized export prices, denominated in U.S. dollars, and obtained the following regression results:

5 The year with the smallest sample size was selected to make the visual display of the pairs of
prices more discernible.

(2) Best estimate: \( \hat{y}_i = 0.983 x_i \); and
(3) Probable range: \( \hat{y}_i = 0.983 x_i \pm 0.091 \).

The regression results suggest that it is not necessary for \( \hat{y}_i = x_i \) (for each \( i = 1 \) to \( N \)) for the CUP to be demonstrated. For example, when the uncontrolled price is at \( x_i = 1.0 \), the paired controlled price may not be \( \hat{y}_i = 1.0 \) exactly. When \( x_i = 1.0 \), \( \hat{y}_i \) may vary from 0.892 to 1.074 (because of random fluctuations) and satisfy the CUP standard. See the development of formula (3) below.

**Fitting Equation with Intercept**

In practice, the regression equation is fitted with an intercept, which in this case is not statistically different from zero. The standard error of the residuals is at \( \sigma = 0.1349 \), \( R^2 = 0.994 \), and \( N = 5,289 \) pairs of price transactions for the year examined. The standard error of the slope coefficient is minuscule at 0.000998, so the linear relationship between the controlled and uncontrolled prices is very strong. Exhibit 1 (shown below) contains all the normalized data points, except for seven outliers.

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6 Upon further examination, the seven outliers set aside from the regression analysis represented product returns.

7 See Theil, Henri, *Economic Forecasts and Policy* (Amsterdam, North-Holland, 1961), pp. 30-37. Theil’s “inequality coefficient” test could be performed, which is related to the root-mean-square test but has a desirable property of being bounded between zero and one.

The regression model herein serves to test if there is a statistical difference between the two sets of prices (i.e., if the pairs of controlled and uncontrolled prices, considered *in toto*, are identical from a statistical perspective). The fitted regression should not be interpreted that \( x \) determines \( y \) as in a typical regression model.

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8 As an alternative, analysts can use correlation analysis to test the relationship between the controlled and comparable uncontrolled prices; however, correlation analysis has several disadvantages compared to regression analysis, and thus has fallen in disfavor. See Wonnacott, Ronald, and Wonnacott, Thomas, *Econometrics* (New York, John Wiley & Sons, 1970), pp. 103-114. The correlation coefficient between \( x_i \) and \( y_i = \sqrt{R^2} = \sqrt{0.994} = 0.997 \), which is very close to one (the maximum value of the correlation coefficient).

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**Arm's-Length Range**

Regs. §1.482-1(e) provides that the arm's-length range is determined by applying a single pricing method, which is selected under the best method rule to two or more uncontrolled transactions of similar comparability and reliability. Regs. §1.482-1(e)(2)(B) provides also that, if exact comparables are not found (which is what usually happens under normal facts and circumstances), the arm's-length range will be determined by including the middle 50 percent of the data. The OECD guidelines are more discretionary, and recognize that "the actual determination of the arm's-length price necessarily requires exercising good judgment." 9

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9 See the OECD guidelines, §1.45. Under this section, the OECD guidelines recognize that "because transfer pricing is not an exact science," there will also be many occasions when the application of the most appropriate method or methods produces a range of figures all of which are relatively equally reliable." In fact, a range of results must be allowed because all measurements are subject to errors irrespective of whether the science is exact or inexact, and the range reflects the standard error of the measurement. The error may be regarded as the sum of two components,
measurement error and stochastic error. See Rescher, Nicholas, *Scientific Explanation* (London, Collier-Macmillan, 1970), especially the Appendix (“Are Historical Explanations Different?”). See also Taylor, John, *An Introduction to Error Analysis* (Oxford University Press, 1982). Taylor contends that:

- every measurement has uncertainties (or is subject to error); and
- it is important to know the magnitude of the uncertainty, because otherwise the result is misleading or inconclusive.

For these reasons, while participating in the drafting of the U.S. temporary and final transfer pricing regulations, the author introduced the idea of allowing ranges of results in all methods testing the arm’s-length consideration of the taxpayer, including the CUP method.

Pursuant to Regs. §1.482-1(e)(2)(A), the arm’s-length range below includes all the comparable uncontrolled prices because the information is sufficiently complete. As a result, the author is confident that all material differences have been identified, and an adjustment was made to eliminate the effect of the difference with respect to daily fluctuations in the commodity’s price.\(^{10}\) In the present case, the arm’s-length range may be calculated according to the approximate formula: \(^{11}\)

\[
\hat{y}_i = 0.983 x_i \pm 0.6745 \sigma; \quad \text{or} \\
\hat{y}_i = 0.983 x_i \pm 0.6745 (0.1349); \quad \text{or} \\
\hat{y}_i = 0.983 x_i \pm 0.091.
\]

Above, \(\hat{y}_i\) denotes the controlled price of the \(i\)-th product predicted by the fitted regression for any given comparable uncontrolled price, \(x_i\) (the CUP for the \(i\)-th product).

For a pedant, the regression results above suggest that \(y_i < x_i\), which means that the foreign parent tended to charge a slightly lower price to its U.S. affiliate than to the uncontrolled distributors (implying, *caeteris paribus*, that the U.S.-related party may have earned a differential profit and paid more tax than the arm’s-length amount owed). However, the small discrepancy can be attributed to random noise, because the regression results show that the controlled and uncontrolled prices are clustered in the same neighborhood.

Based on the statistical evidence analyzed, the taxpayer cannot be subject to a transfer pricing adjustment for this year under examination, a result accepted by the IRS in audit defense. \(^{12}\)

\(^{10}\) See Regs. §1.482-1(e)(2)(iii)(A), which addresses comparables included in the arm’s-length range.

\(^{11}\) A fastidious analyst may find the exact formula for a prediction interval, *inter alia*, in Wonnacott, Ronald and Wonnacott, Thomas, *opere citato*, p. 31 (formula 2.44). The standard error multiplier at \(t = 0.6745\) corresponds to a large sample size \(t\)-statistics, computed at the 50 percent level of confidence.

\(^{12}\) See Regs. §1.482-1(e)(1), which read as follows: “A taxpayer will not be subject to adjustment if its results fall within such arm’s length range.” The scatter diagram exhibiting a near 45-degree line, such that the slope coefficient of the diagonal line is near one, should be sufficient to establish a CUP.

**Conclusion**

In this article, a simple regression model was used to test the existence of comparable uncontrolled prices and proved extremely useful, especially when dealing with a large number of individual transactions. The regression model in equation (1) represents the price charged by the taxpayer to a controlled distributor for the sale of a tangible product as being proportional to the price charged by the taxpayer for the same product in a comparable uncontrolled transaction in a given tax year under review.

The crucial step of the procedure consists in testing whether the estimated slope coefficient (or the factor of proportionality) is within a specified statistical interval including one. If the estimated slope coefficient is within the specified interval, it means that prices charged in controlled and comparable uncontrolled sales are not statistically different. In practice, the model does not establish any causal relationship between the two sets of prices.
The regression method presented above was applied to a complex tax case involving sales from a foreign parent to a controlled U.S. distributor and to several uncontrolled U.S. distributors. Prior to applying the regression model specified under formula (1) above, all prices were adjusted for differences in the date of shipping by scaling them with the corresponding spot price of the commodity (a basic input component of the products exchanged). The statistical results are comforting, with the slope coefficient being very close to one and displaying a minuscule standard error. The CUP analysis demonstrated that the prices charged by the foreign taxpayer to the controlled U.S. distributor was at arm's length and did not justify a tax deficiency, a conclusion that was accepted by the IRS in audit defense.

Appendix

The regression analysis requires an equal number of transactions for the controlled and uncontrolled parties to test the statistical identity between the two prices being compared. Herein, the author provides a solution to the practical problem of the foreign parent having invoiced different numbers of transactions to the controlled and the uncontrolled distributors. For this purpose, the author considered three exhaustive scenarios, where \( N_c \) denotes the number of controlled transactions and \( N_u \) denotes the number of comparable uncontrolled transactions available in the invoice database of the taxpayer under review:

**Case 1.** If \( N_u > N_c \), then reduce \( N_u \) such that \( N_u = N_c \). To avoid biased results, the reduction in the sample size of the comparable uncontrolled transactions must be made by random selection.

**Case 2.** If \( N_u < N_c \), then increase \( N_u \) such that \( N_u = N_c \). Again, to avoid biased results, the increase in the sample size of the comparable uncontrolled transactions must be made by random selection. In this case, the original sample of comparable uncontrolled transactions is used as the sample frame (or as the source of the random drawing of additional observations, where the drawing is made with replacement).

**Case 3.** If \( N_u = N_c \), there is a happy coincidence, and no sampling is required.